



Analysis of Two Dimensional Heat Transfer Problem Using Spreadsheet Approach

A.A Oladosu, I.F Odesola and A.S Olawore

Department of Mechanical Engineering, University of Ibadan, Nigeria

ABSTRACT

The storage of thermal energy in the form of sensible and latent heat has become an important aspect of energy management with the emphasis on efficient use and conservation of the waste heat and solar energy in industry and buildings. Heat storage materials absorb heat through standard heat transfer mechanisms, e.g., radiation, conduction, and convection. The need for developing software for solving these problems become imperative because it has necessary functions for solving a large class of engineering problems. The aim of this project is to demonstrate how Spreadsheet can be used for solving heat transfer problems in an arbitrary geometry. Survey was carried out on various geometries. An example considered was a chimney. The symmetry of the geometry was considered. Energy balances were obtained and finite difference formulations were made. Finite difference equations for various nodes were determined. Spreadsheet was set up for the problem. Various temperature distributions were obtained. Temperatures are highest at the inner wall (but less than 300°C) and the lowest at the outer wall (but more than 260 K), as expected. The average temperature at the outer surface of the chimney was 352.65K. The rate of heat loss through the 1m long section of the chimney was determined both manually and numerically.

Key words: Macro, Spreadsheet, Steady Heat Transfer, Two-dimensional

1.0 INTRODUCTION

Microsoft excel is a very versatile software that has been used far and wide for business calculations. However, lesser use of it has been harnessed in engineering applications. Unfortunately, people are familiar with the use of programming languages such as BASIC, FORTRAN e.t.c. which are cumbersome to operate compare to Microsoft Excel which nevertheless has not well been fully utilized in the field of engineering most especially as it applies to heat transfer. Microsoft Excel, also known as spreadsheet, consists of a 2-D grid which is quite suitable to solve one and two dimensional, steady and transient heat conduction problems.

Many attempts were made to utilize spreadsheets in solving engineering problems. Eid (1987) demonstrated a methodology and a tutorial that described how nodal equations would be written in a Spreadsheet. He derived the nodal equations using two common methods of heat transfer analysis. Baughn and Rossi (1992) used Spreadsheets to solve a two-dimensional transient heat conduction problem through



explicit nodal equations by thermal resistance and capacitance formulations. They used different sets of boundary conditions that included a specified surface temperature, insulated surface, convective heat transfer coefficient and an equivalent radiation heat transfer coefficient. O'Neal [3] used a spreadsheet to analyze a steady temperature distribution in a fin and a one-dimensional transient heat conduction problem. Two dimensional heat conduction was also analyzed using spreadsheets by Thompson [4] along with a one-dimensional transient problem. Weston [5] used spreadsheets to study two and three-dimensional steady heat conduction problems and a transient one-dimensional solution.

Kharab [6] analyzed a transient two-dimensional heat conduction problem using the Lotus 123 spreadsheet and presented the user-macro he used in his analysis. Mokheimer *et al.* [7] used spreadsheets to model steady heat transfer in a composite fin where they solved the problem both analytically and numerically and showed how close spreadsheet numerical solutions are to the analytical solutions. Fisher [8] applied spreadsheets to obtain a graphical solution of equations involving combined modes of heat transfer. He considered a steady one-dimensional problem to analyze heat loss through a multi-layer autoclave with its surface subjected to convection and radiation. Other than using spreadsheets for solving heat transfer problems, they were used for calculating the frequency response of feedback control systems by Wong [9]. Kerkes [10] used spreadsheets to model the transient fluid flow in a porous medium with a moving drainage boundary. Antar [11] used spreadsheets to analyze numerically fluid flow and heat transfer problems that employ the similarity solution.

2.0 MODELING IN HEAT TRANSFER

The descriptions of most scientific problems involve expressions that relate the changes in some key variables to each other. Usually the smaller the increment chosen in the changing variables, the more general and accurate the description. In the limiting case of infinitesimal or differential changes in variables, we obtain differential equations that provide precise mathematical formulations for the physical principles and laws by representing the rates of changes as derivatives. Therefore, differential equations are used to investigate a wide variety of problems in sciences and engineering, including heat transfer.

The study of physical phenomena involves two important steps. In the first step, all the variables that affect the phenomena are identified, reasonable assumptions and approximations are made, and the interdependence of these variables is studied. The relevant physical laws and principles are invoked, and the problem is formulated mathematically. In the second step, the problem is solved using an appropriate approach, and the results are interpreted.



When selecting heat transfer equipment, it is important to consider the actual operating conditions. The heat exchanger must be selected on the basis of operation under these adverse conditions instead of under new conditions. Preparing very accurate but complex models is usually not so difficult. But such models are not much use to an analyst if they are very difficult and time consuming to solve. At the minimum, the model should reflect the essential features of the physical problem it represents. There are many significant real world problems that can be analyzed with a simple model. But it should always be kept in mind that the results obtained from an analysis are as accurate as the assumptions made in simplifying the problem.

A solution that is not quite consistent with the observed nature of the problem indicates that the mathematical model used is too crude. In that case, a more realistic model should be prepared by eliminating one or more of the questionable assumptions. This will result in a more complex problem that, of course, is more difficult to solve. Thus any solution to a problem should be interpreted

2.1 Problem Formulations

The problem considered in this analysis is a two-dimensional steady heat conduction problem.

Heat is transferred in a 2-D system of length L , width W

The governing differential equation can be found in heat transfer and is given as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad (1)$$

The above equation is a second order partial differential equation of the second order and it is otherwise called Laplace equation.

2.1.1 Boundary and Initial Conditions

The heat conduction equations above were developed using an energy balance on a differential element inside the medium, and they remain the same regardless of the *thermal conditions* on the *surfaces* of the medium. To describe a heat transfer problem completely, *two boundary conditions* must be given for *each direction* of the coordinate system along which heat transfer is significant. Therefore, we need to specify *two boundary conditions* for one-dimensional problems, *four boundary conditions* for two-dimensional problems, and *six boundary conditions* for three-dimensional problems. In the case of the wall of a house, for example, we need to specify the conditions at two locations (the inner and the outer surfaces) of the wall since heat transfer in this case is one-dimensional. But in the case of a parallelepiped, we need to specify six boundary conditions (one at each face) when heat transfer in all three dimensions



Yet we know that the heat flux and the temperature distribution in a medium depend on the conditions at the surfaces, and the description of a heat transfer problem in a medium is not complete without a full description of the thermal conditions at the bounding surfaces of the medium. The *mathematical expressions* of the thermal conditions at the boundaries are called the boundary conditions.

2.2 NUMERICAL METHOD IN HEAT CONDUCTION

Due to the increasing complexities encountered in the development of modern technology, analytical solutions usually are not available. For these problems, numerical solutions obtained using high-speed computer are very useful, especially when the geometry of the object of interest is irregular, or the boundary conditions are nonlinear. In numerical analysis, three different approaches are commonly used: the finite difference, the finite volume and the finite element methods.

2.3 TWO-DIMENSIONAL STEADY HEAT CONDUCTION

We will consider the numerical formulation and solution of two-dimensional steady heat conduction in rectangular coordinates using the finite difference method. The approach presented below can be extended to three-dimensional cases.

Consider a *rectangular region* in which heat conduction is significant in the x - and y -directions. Now divide the x - y plane of the region into a rectangular mesh of nodal points spaced Δx and Δy apart in the x - and y -directions respectively, as shown in the Figure below, and consider a unit depth of $\Delta z = 1$ in the z -direction.

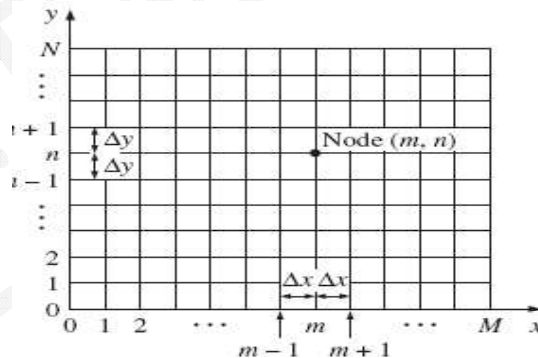


Figure 1 The nodal network for the finite difference formulation

Our goal is to determine the temperatures at the nodes, and it is convenient to number the nodes and describe their position by the numbers instead of actual coordinates. A logical numbering scheme for two-dimensional problems is the *double subscript notation* (m, n) where $m = 0, 1, 2, \dots, M$ is the node count in



the x -direction and $n = 0, 1, 2, \dots, N$ is the node count in the y -direction. The coordinates of the node (m, n) are simply $x = m\Delta x$ and $y = n\Delta y$, and the temperature at the node (m, n) is denoted by $T_{m,n}$.

Now consider a *volume element* of size $\Delta x = \Delta y = 1$ centered about a general interior node (m, n) in a region in which heat is generated at a rate of g and the thermal conductivity k is constant, as shown in Figure below. Again assuming the direction of heat conduction to be *toward* the node under consideration at all

surfaces, the energy balance on the volume element can be expressed as $\left(\begin{array}{l} \text{Rate of heat} \\ \text{generation inside} \\ \text{the element} \end{array} \right) = \left(\begin{array}{l} \text{Rate of change of} \\ \text{the energy content} \\ \text{of the element} \end{array} \right) + \left(\begin{array}{l} \text{Rate of heat conduction} \\ \text{at the left, top, right,} \\ \text{and bottom surface} \end{array} \right)$

$$+ \left(\begin{array}{l} \text{Rate of heat} \\ \text{generation inside} \\ \text{the element} \end{array} \right) = \left(\begin{array}{l} \text{Rate of change of} \\ \text{the energy content} \\ \text{of the element} \end{array} \right) \quad (2)$$

$$\text{Or } \dot{Q}_{\text{cond, left}} + \dot{Q}_{\text{cond, top}} + \dot{Q}_{\text{cond, right}} + \dot{Q}_{\text{cond, bottom}} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta \tau} = 0 \quad (3)$$

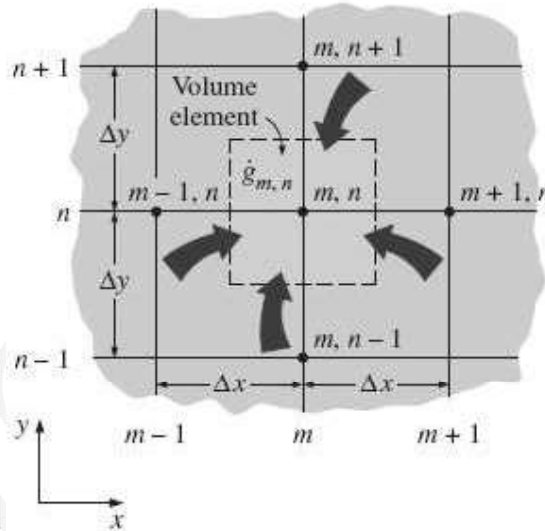


Figure 2 The volume element of a general interior node (m, n) for two-dimensional conduction in rectangular coordinates

For the *steady* case. Again assuming the temperatures between the adjacent nodes to vary linearly and noting that the heat transfer area is $A_x = \Delta y \times 1 = \Delta y$ in the x -direction and $A_y = \Delta x \times 1 = \Delta x$ in the y -direction, the energy balance relation above becomes

$$\frac{T_{m-1,n} - 2T_m + T_{m+1,n}}{\Delta x^2} + \frac{T_{m,n-1} - 2T_m + T_{m,n+1}}{\Delta y^2} + \frac{g_{m,n}}{k} = 0 \quad (4)$$



for $m = 1, 2, 3, \dots, M-1$ and $n = 1, 2, 3, \dots, N-1$. Again a rectangular region M equally spaced nodes in the x -direction and N equally spaced nodes in the y -direction has a total of $(M-1)(N-1)$ nodes, and Equation above can be used to obtain the finite difference equations at all interior nodes. In finite difference analysis, usually a **square mesh** is used for simplicity (except when the magnitudes of temperature gradients in the x - and y -directions are very different), and thus Δx and Δy are taken to be the same. Then $\Delta x = \Delta y = l$, and the relation above simplifies to

$$T_{m-1,n} + T_{m+1,n} + T_{m,n-1} + T_{m,n+1} - 4T_m + \frac{\dot{q}_{m,n} l^2}{k} = 0 \quad (5)$$

That is, the finite difference formulation of an interior node is obtained by adding the temperatures of the four nearest neighbors of the node, subtracting four times the temperature of the node itself, and adding the heat generation term. It can also be expressed in this form, which is easy to remember:

$$T_{left} + T_{top} + T_{right} + T_{bottom} - 4T_{node} + \frac{\dot{q}_{node} l^2}{k} = 0 \quad (6)$$

When there is no heat generation in the medium, the finite difference equation for an interior node further simplifies to $T_{node} = (T_{left} + T_{top} + T_{right} + T_{bottom})/4$ which has the interesting interpretation that the temperature of each interior node is the arithmetic average of the temperatures of the four neighbouring nodes.

For steady 2-dimensional heat transfer the following are the assumptions;

- The rate of change of heat ($\dot{q}$) through the plate is zero i.e. $\dot{q} = 0$
- The rate of change of temperature i.e. $(\frac{\partial T}{\partial t}) = 0$
- Since we are focusing on 2-dimensional heat transfer, then “z” has no place. Therefore, $\frac{\partial T}{\partial z} = 0$

After substituting the above the result gives a homogeneous equation called Laplace equation, which is a partial differential equation of the 2nd order.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad (7)$$

3.0 METHODOLOGY

The focus of this study is to estimate the heat loss in an arbitrary geometry like chimney. A chimney is a hollow vertical structure, usually made of brick or steel that allows gas, smoke, or steam from fire or furnace to escape to the atmosphere.

The following procedures were adopted;



- One-eighth of the geometry whose nodal network consists of nine equally spaced nodes was considered.
- Energy balances were carried out to obtain finite difference formulations.
- Assuming all heat is transferred into volume element for convenience and the formulas for the interior nodes, the finite difference for the nine nodes were determined.
- Spread sheet was set up for the problem.

The schematic diagram of chimney is depicted below.

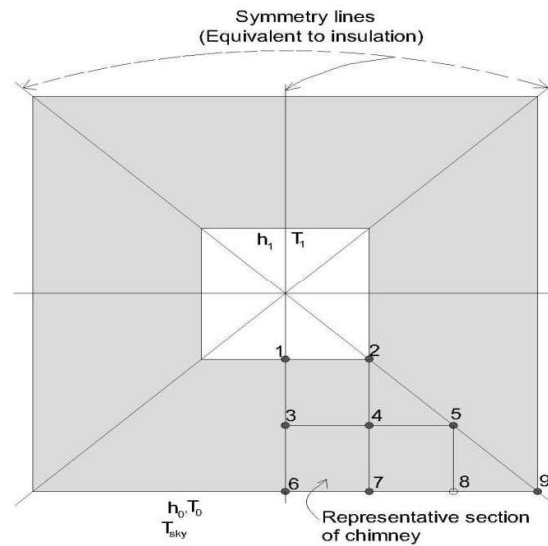


Figure 4 Schematic diagram of the chimney

The data for chimney was given below:

Table 1. Data collected for chimney

Properties	Values
Thermal conductivity (k)	1.4W/m °C
Flow section	20 cm x 20cm
Thickness of the wall	20cm
Average interior temperature of gas (T_i)	300°C
Average convection heat transfer coefficient (h_i)	70W/m ² °C
Average outer surface temperature (T_o)	20°C
Outer heat transfer coefficient (h_o)	21W/m ²



Emissivity of the outer surface Temperature	0.9
Effective sky temperature	260K
$\Delta x = \Delta y$	5cm

3.1 ASSUMPTIONS

Heat transfer is given to be steady and two-dimensional since the height of the chimney is large relative to its cross-section, and thus heat conduction through the chimney in the axial direction is negligible. It is tempting to simplify the problem further by considering heat transfer in each wall to be one dimensional which would be the case if the walls were thin and thus the corner effects were negligible. This assumption cannot be justified in this case since the walls are very thick and the corner sections constitute a considerable portion of the chimney structure. The thermal conductivity is constant.

3.2 SPREADSHEET SOLUTION FOR CHIMNEY

Writing the FDEs for the governing equation for each node within the discretized domain of solution results in $(M + 1) (N + 1)$ algebraic equations. These equations need to be solved simultaneously. The solution of this set of equations is obtained using the *Gauss–Seidel iterative* method built in the spreadsheet programs. So, once all the proper FDEs formula is typed and the boundary conditions are properly imposed, the iterative function should be activated as explained before. Hence, the steady-state solution will be readily evaluated at each cell (node) of the solution range.

Obtaining the final solution, one can use any of the numerous 2-D and 3-D graphical options that are built-in in the spreadsheet programs. Contours, as well as XY-plots, can readily be made and hence the trends are evident right there. A parametric study can be carried out easily using spreadsheet programs as indicated earlier.

4.0 RESULTS AND DISCUSSION

The results of the software were obtained by preparing and running some of the nodal equations that were input into respective cells. Accuracy of the solution can be increased by decreasing the increments in X , Y directions (i.e. by increasing the number of nodes in the numerical mesh), by tightening the convergence criteria in the built-in solver that employs Gauss–Siedel technique in the spreadsheet and by tightening the steady-state criteria in the user-defined-macro that performs the steady solution of the problem. The built-in graphics capabilities of the spreadsheet programs made easy the graphical presentation of the results in



the form of two- or three-dimensional plots, surface plots and contour plots through following a certain graphics wizard.

This wizard guides the user through the whole process starting from defining the range that will be plotted through the various details that controls the type of graph, legends, titles, scales, etc. in an easy to follow procedure.

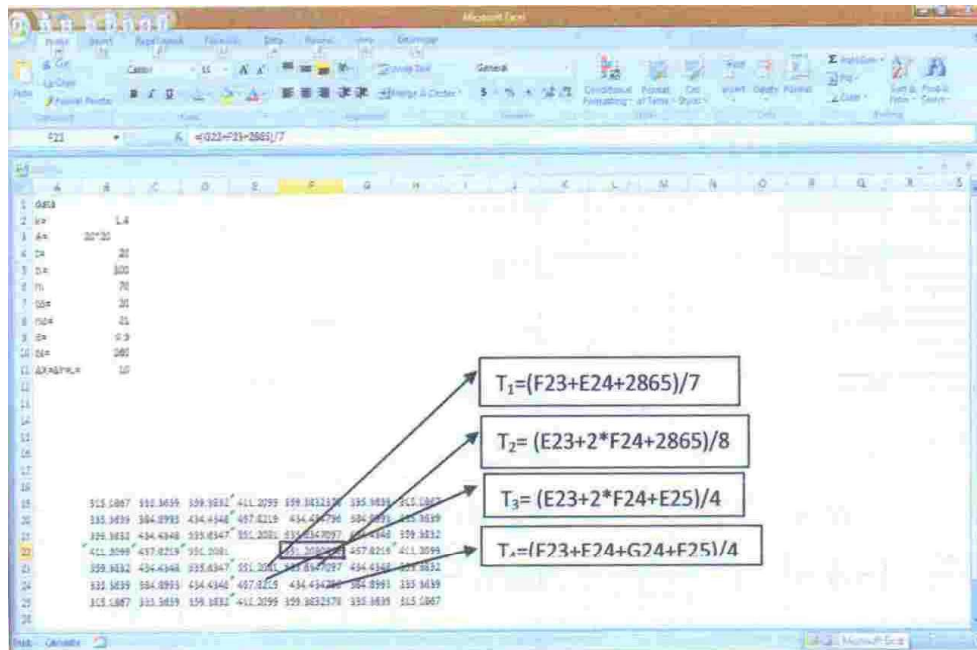


Figure 5. Spreadsheet set up and solution for steady 2-D heat conduction.

The variation of temperature in the chimney is shown in the figure above. The rest of the equations were entered up to $(T_1 - T_9)$, see figure 5 and copied in line with the symmetry of the chimney. These equations were inserted into the corresponding spreadsheet cells.

The temperatures at different nodes are given in the table below:

Table 2 Nodal temperature at one section of the chimney

Nodal temperatures	Temperature (kelvin)
T_1	551.208
T_2	535.634
T_3	457.822
T_4	434.435
T_5	384.889
T_6	411.210
T_7	359.383



T_8	335.364
T_9	315.187

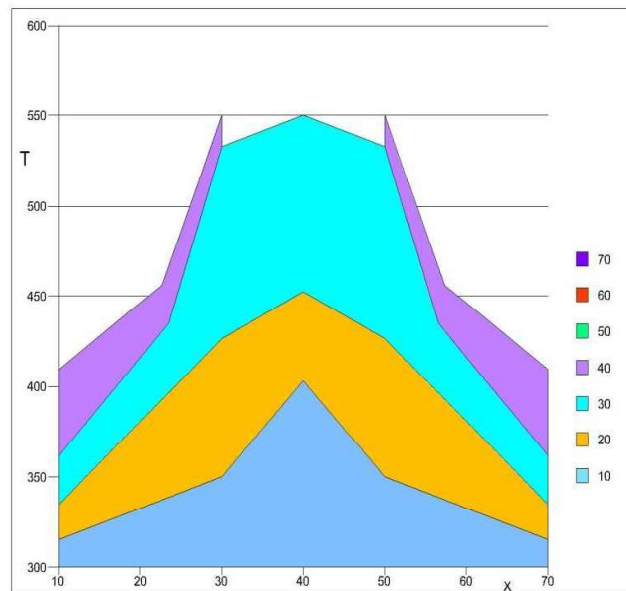


Figure 4 temperature distributions using surface plot

The graphical illustration showing different temperature distributions is shown above (Fig.4). Temperatures in the inner wall are high (but less than 300°C) and lower at the outer wall, as expected. The average temperature at the outer surface of the chimney weighted by the surface area is 352.65K . The total heat transfer can be determined by applying Newton's law of cooling at the surface. The rate of heat loss through the 1m long section of the chimney was determined both manually and numerically. The results gave 1796.5W and 1672W using numerical method and approximate method respectively.

5.0 CONCLUSION

Microsoft Excel was used to solve steady heat transfer problem with constant boundary temperatures. The Spreadsheet was used for solving heat transfer problems in an arbitrary geometry. Numerical solutions were formulated via finite difference method. Graphs were plotted showing various temperature distributions in two dimensions. The results gave 1796.5W and 1672W using numerical method and approximate method respectively. The difference between the two results was due to approximate nature of numerical analysis. The model predicted shows a high level of accuracy and could be a veritable tool in solving heat and other engineering problems. The energy is stored at a relatively constant temperature and



energy losses to the surroundings are lower than with conventional systems. In response to increasing electrical energy costs and the desire for better load management, thermal storage technology has recently been developed. The storage of thermal energy in the form of sensible and latent heat has become an important aspect of energy management with the emphasis on efficient use and conservation of the waste heat and solar energy in industry and buildings. The result was simply presented by using EXCEL interphase.

REFERENCES

1. Arpaci, V. S. and Larsen, P. S., 1994. *Convection Heat Transfer*, Prentice-Hall, Englewood Cliffs, NJ,
2. Baughn, J. W., and Rossi, M., 1992. 'Two-dimensional transient heat conduction analysis using spreadsheets', *Heat Transfer Engineering*, **13**(2), 71–79.
3. Eid, J. E., 1987. A methodology and tutorial for thermal modeling with PC spreadsheets', *Heat Transfer Engineering*, **8**(2), 95–110.
4. K. Wark and D.E Richards, 1999. *Thermodynamics*. 6th ed. New York: Mc Graw-Hill,
5. Kharab, A., 1995. 'Use of a spreadsheet program in a two-dimensional heat conduction problem', *Computer Methods in Applied Mechanics and Engineering*, **122**, 173–181.
6. Lapidus, L. and Pinder, G. F., 1982. *Numerical Solution of Partial Differential Equations in Science and Engineering*, John Wiley & Sons, New York.
7. Mokheimer, E. M. A., Antar, M. A., Farooqi, J., and Zubair, S. M., 1997. 'Analytical and numerical solutions along with PC spreadsheet modeling for a composite fin', *Warme und Stoffubertraguing*, Vol. 32, 229–238.
8. Mokheimer, E. M. A., Antar, M. A., Farooqi, J., and Zubair, S. M., 'Spreadsheet modeling for transient heat conduction in composite fins', Submitted for publication.
9. Ozisik, M. N., 1980. *Heat Conduction*, John Wiley & Sons, New York,.
10. Weston, K. C., 1988. 'Heat transfer simulation using productivity tools', Proc. Summer Computer Simulation Conf. of the Society for Computer Simulation, pp. 260–265.