

Solar thermal radiation and Cattaneo–Christov heat transport in magnetohydrodynamic photovoltaic-thermal cooling systems under rotating flow

Leila Manai ^{*,§}, M. J. Johnson [†], Saleh Chebaane ^{*} and A. M. Obalalu [‡]

^{*}*Department of Physics, College of Science,
University of Ha'il, P.O. Box 2440, Ha'il, Saudi Arabia*

[†]*Department of Physics and Materials Science,
Kwara State University, Malete, Nigeria*

[‡]*Department of Mathematics and Statistics,
Kwara State University, Malete, Nigeria*

[§]*lem.manai@uoh.edu.sa; manailaila123@gmail.com*

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The integration of photovoltaic-thermal (PVT) systems with advanced cooling techniques is crucial for improving energy efficiency and overcoming performance limitations in solar energy conversion. This study investigates the combined effects of solar thermal radiation, Cattaneo–Christov non-Fourier heat transport and magnetohydrodynamic (MHD) rotating flow on PVT cooling performance. The purpose is to provide a deeper understanding of how thermal and electromagnetic interactions influence heat transfer and system efficiency, particularly under conditions where conventional Fourier-based models fall short. A carboxymethylcellulose (CMC)–water-based base fluid enhanced with nanoparticles zirconium oxide (ZrO_2), copper (Cu) and aluminum oxide (Al_2O_3) is employed as the working medium to further boost thermal conductivity and system efficiency. A mathematical model is developed to capture the dynamics of radiative heat flux, magnetic field, rotation and nonlinear thermal relaxation effects. The governing equations are transformed into dimensionless form and solved numerically using a Chebyshev collocation method. The results demonstrate that the inclusion of ZrO_2 , Cu and Al_2O_3 nanoparticles in the CMC–water base fluid significantly improves thermal conductivity, thereby enhancing the system's cooling capacity. Under solar thermal radiation, ternary hybrid nanofluids demonstrate approximately 20% superior thermal regulation. The novelty of this work lies in the synergistic analysis of MHD rotating flow, Cattaneo–Christov heat transport and nanoparticle-engineered CMC-based fluids for PVT cooling, providing new physical insights and practical guidelines for the design of high-performance renewable energy systems.

Keywords: Photovoltaic-thermal (PVT); ternary hybrid nanofluid; magnetohydrodynamic (MHD); solar thermal radiation.

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[§]Corresponding author.

1. Introduction

Photovoltaic thermal (PVT) are devices that integrate a photovoltaic (PV) array with a thermal collector to generate electricity and provide thermal energy by using solar radiation.¹ PVT systems utilize waste heat, unlike stand-alone PV systems, which convert sunlight into electricity with efficiencies typically ranging from 15% to 20%, while most of the absorbed energy is lost as heat. By harnessing this heat, PVT systems achieve higher overall energy efficiency and greater energy output for the same system size.² The basic concept relies on the physical principles of solar energy conversion, heat transfer and fluid dynamics, in which a working fluid (e.g., water, air or nanofluids) circulates through a thermal collector attached to the PV module to remove heat, cool the PV cells and improve their electrical efficiency while simultaneously generating useful thermal energy.³ PVT systems are typically modeled as a system of coupled heat transfer, fluid flow and PV performance equations, frequently with more complex physics and materials included, such as thermal radiation, convection and PV material characteristics.⁴ Designing PVT systems is an interdisciplinary area touching renewable energy, thermal sciences, as well as material sciences, which means that PVT systems offer a potential solution to sustainable energy generation. PVT systems are also essential, as they provide the largest solar energy utilization rate in response to the rising demands of effective and renewable sources of energy.⁵ PVT systems, or the integration of electricity and heat production, enable overall efficiencies (up to 70–80% in certain designs) that are higher than either a stand-alone PV or solar thermal subsystem. Applications include residential and commercial applications in which PVT systems supply electricity and hot water to heat a building, cool it or serve some industrial process. In agriculture, they aid greenhouse heating and drying processes, and in cities, they aid building integral energy solution, where there is less dependence on fossil fuels. New applications are coming forth that involve the combining of the application with district heating, with desalination plants or with other hybrid energy systems, enhancing PVT and heat pumps, or energy storage.⁶ With increased energy efficiency and lowered carbon footprints, PVT systems serve as an integral part of sustainable development and in the fight against climate change.⁷ PVT systems research is broad in extent, covering experimental design, theoretical modeling and computational optimization. To develop better efficiency in the system, researchers have been toying with the idea of exploring better materials, selective coatings or phase-change materials, and new materials used as a working fluid, such as nanofluids and ternary hybrid nanofluids, to enhance heat movement. There is also research about the use of systems configuration, e.g., air-based, liquid-based or concentrated PVT systems, or a combination of systems with energy storage or smart grids.⁸ PV performance is mathematically linked to equations describing thermofluids and heat transfer, sometimes with notions such as buoyancy, thermal radiation or magnetohydrodynamic (MHD) effects included in more advanced designs. Simulation system performance in different environmental conditions is performed with the help of such

computing tools as finite element methods and computational fluid dynamics (CFD). The problem is not over, though. Materials weaken when exposed to intense temperatures and UV rays, thus making it hard to design cost-efficient and long-lasting PVT systems.⁹ To achieve optimality in terms of heat transfer without loss of electrical efficiency, there must be trade-offs, i.e., protecting the PV cells with many coolant fluid flows and flows that balance the heat transfer. Large-scale implementation of the PVT systems presents economic and manufacturing challenges, and the use of big physical networks with varied physical contacts cannot utilize simple computational resources.¹⁰ Measured experiments are also difficult to validate because solar input and environmental variables are variable. Although these barriers exist, continuous development of PVT systems stands to innovate, posing huge promises of increased application of renewable energy in different settings.¹¹ In an attempt to further ascertain the viability of PVT and concentrated PVT systems as some of the possible solutions to sustainable energy, Roudbari *et al.*¹² provided a detailed overview of the current trends in these new technologies. It is concluded in the review that innovation and advances in cost-effectiveness remain essential to wider implementation, and PVT technology is a serious contender as part of clean energy infrastructure in the future and sustainable efforts worldwide. Rajamony *et al.*¹³ studied the PVT systems combined with phase change materials (PCMs) to control the temperature of the system. However, the study examines integration frameworks of PCM, dust mitigation solutions and technological developments to promote the efficiency of PVT. Kazem *et al.*¹⁴ discussed their sensitivity to urban setting restrictions on renewable energy with the use of building facades with building-integrated photovoltaic/thermal (BIPV/T) systems. Such systems produce electricity and thermal energy simultaneously, combining the high energy requirement and emissions caused by buildings.

Thermal radiation is a fundamental mode of heat transfer that occurs due to the emission of electromagnetic waves from the surface of a body as a result of its temperature. Unlike conduction and convection, which require physical contact or a medium, thermal radiation can propagate through a vacuum, making it a dominant mechanism of heat transfer in space and high-temperature environments.¹⁵ It originates from the random thermal motion of charged particles, primarily electrons, within atoms and molecules, leading to the emission of photons across a wide spectrum. The amount, direction and spectral characteristics of the radiated energy are governed by the laws of radiation, such as Planck's law, Stefan–Boltzmann law and Kirchhoff's law of thermal radiation.¹⁶ The study of thermal radiation is of immense importance in both natural and engineered systems. In astrophysics and atmospheric science, it explains phenomena such as the heat balance of planets, the greenhouse effect and the radiation emitted by stars. In engineering, thermal radiation plays a crucial role in the design of solar collectors, PVT systems, furnaces, combustion chambers and thermal insulation materials.¹⁷ It is equally vital in space technologies, where it governs thermal management of satellites and spacecraft operating in vacuum conditions. Moreover, advanced applications such as radiative cooling,

thermal imaging, infrared sensing and energy-efficient building design heavily rely on controlling and optimizing radiative heat transfer.¹⁸ Sharma *et al.*¹⁹ investigated how solar thermal radiation affects the flow behavior of Maxwell nanofluids over a permeable stretching sheet using the Bvp4C method. Their study specifically examines the roles of activation energy and heat radiation in this dynamic system. Eswaramoorthi *et al.*¹⁸ used numerical methods to study how solar heat affects an unsteady, electrically conducting and reactive fluid flow. They applied the second law of thermodynamics and considered the effects of thermophoresis (the movement of particles due to temperature differences). Zhiying *et al.*²⁰ studied the importance of directional thermal radiation and how it is useful in enhancing the performance of thermal gadgets by reducing the nondirectional loss of energy during passing. Their research compares conventional isotropic radiation to directional thermal emission and discusses how it is vital in various uses like thermal imaging, radiation, infrared encryption and energy management.

The quest for efficient thermal management has long been a central focus in engineering, energy systems and industrial applications. Conventional heat transfer fluids such as water, ethylene glycol and oils have inherent limitations in thermal conductivity, which restrict their effectiveness in advanced cooling and heating processes.²¹ To overcome these limitations, researchers have explored new classes of working fluids that can provide superior thermal properties. In this context, nanofluids — engineered colloidal suspensions of nanometer-sized particles (1–100 nm) in base fluids — have emerged as a promising innovation. The concept was first introduced in the mid-1990s and since then, nanofluids have attracted significant attention due to their ability to enhance thermal conductivity, improve heat transfer rates and optimize energy utilization.²¹ The enhanced properties of nanofluids are attributed to the high surface area-to-volume ratio of nanoparticles, Brownian motion and interfacial layering effects, which collectively improve the overall thermal transport mechanisms. Nanofluids hold great importance in bridging the gap between conventional fluids and high-performance thermal systems.²² Their superior thermal properties make them valuable in applications where efficient heat transfer is critical. In renewable energy systems, such as PVT collectors and solar concentrators, nanofluids improve energy conversion efficiency by enhancing heat absorption and transfer.²³ In the automotive and aerospace sectors, they are used in the cooling of engines, electronic devices and high-performance batteries, ensuring system stability and longevity. Nanofluids are also employed in medical applications, particularly in hyperthermia treatment for cancer therapy, targeted drug delivery and biomedical imaging, where precise heat control is essential. Additionally, they are used in nuclear reactors, microelectronics cooling and industrial processes involving heat exchangers, boilers and chillers. The versatility and adaptability of nanofluids across diverse fields underscore their significance as next-generation working fluids for advancing energy efficiency, sustainability and technological innovation.²⁴ Lone *et al.*²⁵ examined the heat transfer enhancement of a water-based ternary hybrid nanofluid consisting of single-walled carbon nanotube, multi-walled

carbon nanotube, graphene oxide nanoparticles over a stretched cylinder. Lone *et al.*²⁶ investigated the thermal and flow performance of a ternary hybrid nanofluid on the porous, stretchy 3D surface in which the effect of thermal radiation, thermophoresis, chemical reactions, Joule heating and convective/mass flux is considered.

MHD is a branch of physics and engineering that studies the behavior of electrically conducting fluids, such as plasmas, liquid metals and electrolytes, under the influence of magnetic and electric fields. The theoretical foundation of MHD was established in the early 20th century, particularly through the pioneering works of Hannes Alfvén, who introduced the concept of Alfvén waves and laid the groundwork for plasma physics.²⁷ The essence of MHD lies in combining the principles of fluid dynamics with Maxwell’s equations of electromagnetism to describe how charged particles in a moving fluid interact with magnetic fields.²⁸ This interaction results in complex phenomena such as magnetic damping, induced currents, and modifications of velocity and pressure distributions, making MHD a fundamental tool for understanding natural and engineered systems. The study of MHD is of great importance because it provides insights into both natural processes and technological innovations.²⁹ In astrophysics and geophysics, MHD explains the dynamics of stellar and planetary magnetic fields, solar wind interactions with planetary atmospheres and the behavior of astrophysical plasmas in galaxies. In engineering, MHD principles are applied to the design of advanced cooling systems for nuclear reactors, MHD generators for direct conversion of thermal energy to electricity, and propulsion systems for space exploration and marine vehicles. Furthermore, MHD plays a vital role in controlled nuclear fusion research, where plasma confinement in magnetic fields is essential for achieving sustainable energy production. Ishak *et al.*³⁰ investigated the steady mixed convection MHD flow at a stagnation point over a vertical surface in a micropolar fluid under forced convection conditions. Hussein *et al.*²⁹ studied the MHD flow and heat transfer of a ternary hybrid nanofluid (copper–alumina–titania/water) over a shrinking/stretching cylinder with velocity slip, curvature and variable wall temperature. The ternary nanofluid exhibited reduced heat transfer and increased skin friction as opposed to copper–alumina/water.

Internal heat generation (or generation of thermal energy in a material or system) involves the generation of thermal energy by internal processes rather than the outer processes of conduction or radiation. These internal processes include chemical reactions and nuclear reactions, electrical resistance or viscous dissipation.²⁹ The phenomenon is important in the theory of fluid dynamics with heat transfer, especially in situations where the flow itself or spatial temperature patterns are caused by self-sustaining sources of heat. Internal heat generation, included as a source term in the governing equations (such as the energy equation in the Navier–Stokes framework), can influence the system’s temperature distribution, alter convection patterns and affect its overall stability. The internal heat generation is useful in fluid mechanics, thermodynamics and materials science, and it is important in modeling complex systems such as biological tissues and industrial reactors. It is important in

nuclear engineering to describe the production of heat in reactors, where fission reactions yield very large amounts of internal energy that need to be controlled in the interests of reactor safety and efficiency. Exothermic reactions, as used in chemical engineering within reactors or combustion systems, are dependent on internal heat generation and influence the rates of reactions and the product yield. It is important in biomedical research as a form of cancer treatment, hyperthermia therapy, in which maintained levels of heat production within tissues are used, which is a particular biomedical application of hyperthermia. Radioactive decay releases internal heat that causes the mantle to conduct with 200 kJ of energy per unit area and affects plate tectonics and volcanic activity in geophysics. Knowledge of internal heat sources is also vital in industrial processes through polymer curing, food preparation and electronics cooling. Internal heat generation with a proper simulation and control modeling of these processes is beneficial in the energy efficiency, system performance and safety in different fields.³¹ The research on internal heat generation spans theoretical and computational issues, as well as experimental investigation into the effects of internal heat generation within a variety of systems. Recent applications in new areas include microfluidics, renewable energy systems and advanced manufacturing, where control of heat production is precise. Fatunmbi *et al.*³² investigated the characteristics of the flow and heat transfer of an MHD fluid of the Burgers over a stretching cylinder with internal heat generation and thermal radiation. Afzal and Kamran³³ conducted a computational investigation of heat and mass transfer in MHD hybrid nanofluid flow over a linearly stretching sheet with internal heat generation. This study by Venkateswarlu *et al.*³⁴ developed the chemical reaction and MHD flow of a Casson nanofluid on a moving wedge, considering such effects as thermal radiation, viscous dissipation and heat generation-absorption internally, under the convective boundary conditions. Optimizing the performance of thermal radiation and nanomaterials for thermophoresis in hybrid nanofluid: applications in advanced thermal management systems was investigated by Ref. 37. Design advanced perspectives of Darcy-Forchheimer flow of thermophoretic particle deposition in hybrid nanofluid capturing thermal radiation and Soret-Dufour effects was studied by Ref. 38. Optimizing the performance of Stefan blowing and nanomaterials for ternary hybrid nanofluid with gyrotactic microbes and convective boundary conditions was examined by Ref. 39. A semi-analytical study considering Christov-Cattaneo flux model and Stephan blowing effects: thermally radiative flow of ternary hybrid nanofluid was explored by Ref. 40. Evaluating the impact of thermal radiation and elastic deformation on $\text{Co}_3\text{O}_4/\text{NaC}_6\text{H}_7\text{O}_6$ -based nanofluid with local thermal nonequilibrium conditions was analyzed by Ref. 41. Optimizing fluid flow and heat transmission in industrial processes with Cattaneo-Christov flux model: The potential of thermal radiation and trihybrid-nanofluid was studied by Ref. 42. Stochastic-based deep neural network for analysis of bioconvection flow and thermal radiation: applications of nanofluid in industrial cooling systems was studied by Ref. 43. Magnetic field modulation of electroosmotic-peristaltic flow in the tumor microenvironment was

investigated by Ref. 44. Magnetic field modulation of electroosmotic-peristaltic flow in the tumor microenvironment was evaluated by Ref. 44. Darcy–Forchheimer flow driven by membrane pumping and MHD: perturbation solution was assessed by Ref. 45. Heat Transfer and Flow Analysis in Tumor-Obstructed Propagating Microchannel was studied by Ref. 46. Melting heat transfer in electro-osmotic augmented stagnation point flow of a micropolar fluid over a stretching porous sheet was considered by Ref. 47. Heat transfer and entropy generation in a microchannel with induced magnetic field and membranes pumping: ghost-valve model was considered by Ref. 48. Membrane-driven flow and heat transfer of viscoelastic fluids: MHD and entropy generation analysis was studied by Ref. 49.

PVT systems are increasingly recognized as promising technologies for maximizing solar energy utilization by generating both electricity and thermal energy. However, their efficiency is often hindered by excessive heat accumulation on the PV surface, which lowers electrical output and compromises system durability. Conventional cooling approaches, largely based on Fourier’s heat conduction model, fall short in capturing the nonlinear thermal relaxation and electromagnetic effects that arise under real PVT operating conditions. Similarly, traditional base fluids with low thermal conductivity limit the cooling capacity and overall performance of these systems. These challenges necessitate advanced thermal management strategies that integrate improved physical models, enhanced heat transfer fluids, and electromagnetic effects to deliver higher efficiency and reliability.

This study addresses these limitations by developing a comprehensive model that combines Cattaneo–Christov heat transport, MHD rotating flow and ternary hybrid nanofluids based on a carboxymethylcellulose (CMC)–water-based fluid enhanced with zirconium oxide (ZrO_2), copper (Cu) and aluminum oxide (Al_2O_3) nanoparticles. The purpose is to provide a deeper understanding of how the interplay between solar thermal radiation, rotation, magnetic fields and nonlinear relaxation effects influences heat transfer and cooling performance in PVT systems. A mathematical framework is established, the governing equations are nondimensionalized, and numerical solutions are obtained using the Chebyshev collocation method to explore system dynamics. The motivation behind this work stems from the urgent need to improve renewable energy technologies for sustainable energy generation. By going beyond traditional Fourier-based models and simple base fluids, this study introduces Cattaneo–Christov heat transport as a more accurate description of non-Fourier thermal relaxation and utilizes MHD rotating flow as an additional mechanism for fluid control.

1.1. Research questions

To achieve the stated aim and objectives, the following research questions guide this study:

- How does the integration of Cattaneo–Christov non-Fourier heat transport improve the accuracy of predicting heat transfer in PVT cooling systems compared to the conventional Fourier model?

- What is the impact of MHD rotating flow on the velocity, temperature and thermal boundary layer characteristics of nanofluid-based PVT cooling systems?
- How do solar thermal radiation effects interact with MHD forces and non-Fourier relaxation time to influence heat transfer efficiency in PVT systems?
- To what extent does the use of ternary hybrid nanofluids ($\text{ZrO}_2\text{-Cu-Al}_2\text{O}_3/\text{CMC-water}$) enhance thermal conductivity and improve system cooling compared to mono- and binary nanofluids?
- What is the comparative performance of the proposed ternary hybrid nanofluids under varying magnetic field strengths, rotation rates and radiation intensities?
- How does the Chebyshev collocation method perform in capturing the nonlinear coupled effects of heat transfer, rotation and radiation in this system, and what advantages does it offer over other numerical techniques?
- Can the incorporation of ZrO_2 , Cu and Al_2O_3 nanoparticles in CMC-water lead to a measurable improvement in PVT system efficiency, and what are the physical mechanisms driving this enhancement?
- What practical design guidelines and optimization strategies for PVT systems can be derived from the combined analysis of non-Fourier heat transport, MHD rotating flow and ternary hybrid nanofluid?

2. Model Assumptions

To examine the MHD flow and thermal transport in a PVT system within a rotating Casson nanofluid environment, the following assumptions are made:

- Flow characteristics:
The fluid motion is considered steady, laminar and incompressible within a two-dimensional domain confined between two infinite, parallel flat plates spaced a distance h apart.
- Plate motion:
The lower boundary at $y = 0$ undergoes a linear shear motion with velocity $Uw \text{ } mx \text{ } (m > 0)$.
The upper plate at $y = h$ either permits or resists fluid through uniform suction or injection at a constant velocity v_0 .
- Rotation:
The entire channel rotates uniformly about the vertical y -axis with angular speed Ω , introducing Coriolis forces into the flow field.
- Magnetic field:
An externally applied constant magnetic field of intensity B_0 acts perpendicularly to the flow in the z -direction. This field induces Lorentz forces that influence the momentum transport in the nanofluid.

- Negligible Hall and ion slip effects:
- Owing to the negligible magnetic field strength and the absence of ionized plasma, the Hall current and ion slip effects are omitted. This is in agreement with previous research, which confirms that these phenomena become relevant primarily in strong magnetic or plasma-dominated environments.^{35,36}
- Exclusion of Hall and ion slip effects:
The ion slip and Hall current phenomena are excluded from the model because the fluid is not highly ionized and the applied magnetic field is relatively weak. Literature supports that such effects are only prominent under intense magnetic conditions or in plasma flows.
- Thermal radiation treatment:
The nanofluid is regarded as optically thick, making the Rosseland diffusion approximation suitable for modeling radiative heat transfer in the energy equation. Figure 1(b) displays a comparative schematic of four advanced solar thermal radiation systems.
- Internal heating:
A uniform internal heat source denoted by Q_0 is assumed to be present throughout the flow domain, contributing additional energy to the thermal system.

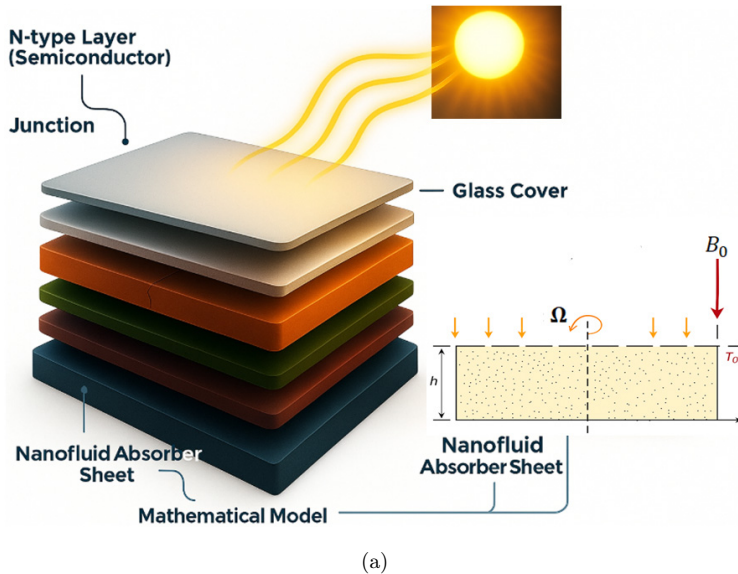
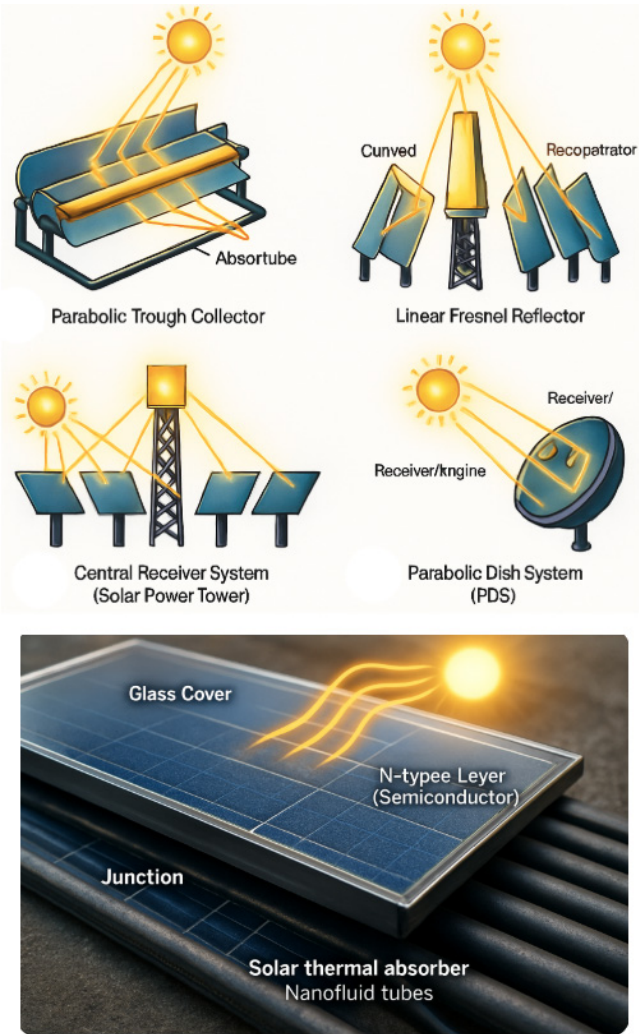


Fig. 1. (Color online) (a) Flow configuration, (b) Four advanced solar thermal radiation systems.



(b)

Fig. 1. (Continued)

- Fluid properties:
The thermophysical characteristics of the nanofluid, such as viscosity, thermal and electrical conductivities, and specific heat are treated as constant and uniform throughout the flow field.
- Neglect of buoyancy:
Gravitational forces and buoyancy effects are considered negligible as the setup involves horizontal flow with no significant vertical temperature gradients.

- Non-Fourier heat conduction:

In order to model thermal wave propagation at a finite speed, the classical Fourier heat conduction law is replaced by the Cattaneo–Christov framework. This is particularly suitable for nanofluids, where thermal relaxation effects are significant due to high conductivity and microstructural interactions.

2.1. Constitutive equation for Cattaneo–Christov heat flux

The Cattaneo–Christov heat flux model generalizes Fourier’s law by introducing a thermal relaxation time, thereby capturing delayed thermal responses in fluids. The upper-convected derivatives of a vector field F (e.g., the heat flux or mass flux vector) in a general coordinate system is

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + V \cdot \nabla F - F \cdot \nabla V + (\nabla \cdot V)F. \quad (1)$$

In this context, V denotes the velocity vector of the fluid. Accordingly, the Cattaneo–Christov model incorporates a finite speed for heat propagation, overcoming the limitation of infinite thermal transmission predicted by Fourier’s law. The heat flux equation is rewritten in modified form as

$$q + \lambda \left(\frac{\partial q}{\partial t} + V \cdot \nabla q - q \cdot \nabla V + (\nabla \cdot V)q \right) = -k \nabla T. \quad (2)$$

In this expression, q is the heat flux vector, λ signifies the thermal relaxation time accounting for the lag in heat flux response, k is the thermal conductivity, and T is the temperature field. This expression takes into account the convective and time variation of heat flux. When $\lambda = 0$, the classical Fourier law is obtained. In the case of an incompressible fluid, the divergence of the velocity becomes ($\nabla \cdot V = 0$) and the equation reduces to

$$q + \lambda \left(\frac{\partial q}{\partial t} + V \cdot \nabla q - q \cdot \nabla V \right) = -k \nabla T. \quad (3)$$

2.2. Governing equations

Continuity Equation:

$$\frac{\partial \check{u}\check{u}}{\partial \check{x}} + \frac{\partial \check{v}}{\partial \check{y}} = 0. \quad (4)$$

Momentum Equations:

$$\check{u} \frac{\partial \check{u}}{\partial \check{x}} + \check{v} \frac{\partial \check{u}}{\partial \check{y}} + 2\Omega \check{w} = -\frac{1}{\rho_{nf}} \frac{\partial p^*}{\partial x} + \frac{\mu_{thnf}}{\rho_{thnf}} \left(1 + \frac{1}{\beta} \right) \left(\frac{\partial^2 \check{u}}{\partial \check{x}^2} + \frac{\partial^2 \check{u}}{\partial \check{y}^2} \right) - \frac{\sigma_{thnf}}{\rho_{thnf}} B_0^2 \check{u}, \quad (5)$$

$$\check{u} \frac{\partial \check{v}}{\partial \check{x}} + \check{v} \frac{\partial \check{v}}{\partial \check{y}} = -\frac{1}{\rho_{thnf}} \frac{\partial p^*}{\partial y} + \frac{\mu_{thnf}}{\rho_{thnf}} \left(1 + \frac{1}{\beta} \right) \left(\frac{\partial^2 \check{v}}{\partial \check{x}^2} + \frac{\partial^2 \check{v}}{\partial \check{y}^2} \right), \quad (6)$$

$$\check{u} \frac{\partial \check{w}}{\partial \check{x}} + \check{v} \frac{\partial \check{w}}{\partial \check{y}} - 2\Omega \check{u} = \frac{\mu_{\text{thnf}}}{\rho_{\text{thnf}}} \left(1 + \frac{1}{\beta} \right) \left(\frac{\partial^2 w}{\partial \check{x}^2} + \frac{\partial^2 w}{\partial \check{y}^2} \right) - \frac{\sigma_{\text{thnf}}}{\rho_{\text{thnf}}} B_0^2 \check{w} \quad (7)$$

Energy Equation:

$$\begin{aligned} \frac{(\rho C_P)_{\text{thnf}}}{\kappa_{\text{thnf}}} & \left(\left(\check{u} \frac{\partial T}{\partial \check{x}} + \check{v} \frac{\partial T}{\partial \check{y}} \right) + \varrho \left(u \frac{\partial \check{u}}{\partial \check{x}} \frac{\partial T}{\partial \check{x}} + \check{v} \frac{\partial \check{u}}{\partial \check{y}} \frac{\partial T}{\partial \check{x}} + \check{u} \frac{\partial v}{\partial \check{x}} \frac{\partial T}{\partial \check{y}} + \check{v} \frac{\partial v}{\partial \check{y}} \frac{\partial T}{\partial \check{y}} \right. \right. \\ & \left. \left. + 2\check{u}\check{v} \frac{\partial^2 T}{\partial \check{x} \partial \check{y}} + \check{u}^2 \frac{\partial^2 T}{\partial \check{x}^2} + \check{v}^2 \frac{\partial^2 T}{\partial \check{y}^2} \right) \right) = \frac{\partial^2 T}{\partial \check{x}^2} + \frac{\partial^2 T}{\partial \check{y}^2} - \frac{1}{\kappa_{\text{thnf}}} \frac{\partial q_r}{\partial \check{y}} + \frac{Q_0(T - T_0)}{\kappa_{\text{thnf}}} \\ & + \frac{\sigma_{\text{thnf}}}{\kappa_{\text{thnf}}} B_0^2 (\check{u}^2 + \check{v}^2) + \frac{\mu_{\text{thnf}}}{\kappa_{\text{thnf}}} \left(1 + \frac{1}{\beta} \right) \left[2 \left[\left(\frac{\partial \check{u}}{\partial \check{x}} \right)^2 + \left(\frac{\partial \check{v}}{\partial \check{y}} \right)^2 \right] \right. \\ & \left. + \left(\frac{\partial \check{v}}{\partial \check{x}} + \frac{\partial \check{u}}{\partial \check{y}} \right)^2 + \left(\frac{\partial \check{w}}{\partial \check{y}} \right)^2 + \left(\frac{\partial \check{w}}{\partial \check{x}} \right)^2 \right]. \end{aligned} \quad (8)$$

2.3. Rotational flow equation

To remove the pressure gradient terms, Eqs. (5) and (6) are differentiated with respect to $\check{y}$ and $\check{x}$, respectively, and subtracted^{35,36}:

$$\begin{aligned} \frac{\partial}{\partial \check{y}} \left(\check{u} \frac{\partial \check{u}}{\partial \check{x}} + \check{v} \frac{\partial \check{u}}{\partial \check{y}} + 2\Omega \check{w} \right) - \frac{\partial}{\partial \check{x}} \left(\check{u} \frac{\partial v}{\partial \check{x}} + \check{v} \frac{\partial v}{\partial \check{y}} \right) - \frac{\sigma_{\text{thnf}}}{\rho_{\text{thnf}}} B_0^2 \frac{\partial \check{u}}{\partial \check{y}} \\ - \frac{\mu_{\text{thnf}}}{\rho_{\text{thnf}}} \left(1 + \frac{1}{\beta} \right) \left[\frac{\partial^3 \check{u}}{\partial \check{x}^2 \partial \check{y}} + \frac{\partial^3 \check{u}}{\partial \check{y}^3} - \frac{\partial^3 \check{v}}{\partial \check{x}^3} - \frac{\partial^3 \check{v}}{\partial \check{x} \partial \check{y}^2} \right] = 0. \end{aligned} \quad (9)$$

This equation describes the rotational balance of the flow field and accounts for rotational forces, magnetic damping and shear interactions in the nanofluid medium.

2.4. Boundary conditions

The boundary conditions for the velocity and temperature fields are specified as

$$\check{u} = U_w = m\check{x}, \quad \check{v} = 0 = \check{w}, \quad T = T_a \quad \text{at } \check{y} = 0 \text{ (Lower wall),} \quad (10)$$

$$\check{u} = 0, \quad \check{v} = -v_0, \quad T = T_0 \quad \text{at } \check{y} = h \text{ (Upper wall).} \quad (11)$$

2.5. Thermal characteristics of nanoparticles and base fluid

Table 1 provides important thermophysical characteristics of a CMC–water-based base fluid and nanoparticles ZrO₂, Cu and Al₂O₃. They are typically studied in the design and optimization of nanofluids used in the PVT system, where heat extraction and heat control are important in enhancing system efficiency^{14,25}

CMC–Water-Based Fluid:

- The CMC–H₂O combination has a density of 997.1 kg/m³ and a high specific heat capacity (4179 J/kg·K), making it a base fluid that can be used with high thermal

Table 1. Thermophysical properties of CMC–water-based base fluid and nanoparticles ZrO_2 , Cu and Al_2O_3 .

Thermophysical attributes	$\rho(\text{kg/m}^3)$	C_p (J/kg·K)	k (W/m·K)	σ (S/m)
CMC– H_2O (0.2–0.4%)	997.1	4179	0.613	5.5×10^{-6}
ZrO_2	5680	502	1.7	4.1×10^{-6}
Copper	8933	385	401	1.587
Al_2O_3	3970	765	30	1×10^{-8}

storage capacity. Its low thermal conductivity (0.613 W/m·K) is a limitation for PVT systems, which is why nanoparticles are introduced to increase its thermal performance. CMC can also add non-Newtonian properties to the fluid, enhancing the stability of the fluid in nanofluid applications.

ZrO_2 :

- ZrO_2 is a ceramic nanoparticle known for its chemical stability and moderate thermal conductivity (1.7 W/m·K). In PVT systems, the addition increases the base-plate thermal conductivity of the working fluid, which allows the improvement of the thermal removal of heat from PV panels without a substantial impact on the electrical conductivity of the system.

Cu:

- Cu is a metal nanoparticle that possesses superior thermal conductivity (401 W/m·K) and high electrical conductivity (5.96×10^7 S/m). It has a high density (8933 kg/m³) and a low specific heat (385 J/kg·K) that make it attractive in applications that require rapid heat dissipation, like electronic cooling and heat exchangers.

Al_2O_3 :

- Al_2O_3 offers a balance between thermal conductivity (30 W/m · K) and electrical insulation (1×10^{-8} S/m). It has a moderate density (3970 kg/m³) and specific heat (765 J/kg K). Hence, it dominates in nanofluids as a means of enhancing thermal performance without compromising on dielectric characteristics.

$$\begin{aligned}
\phi_1 &= \phi_{\text{ZrO}_2}, \quad \phi_2 = \phi_{\text{Cu}}, \quad \phi_3 = \phi_{\text{Al}_2\text{O}_3}, \\
\mu_{\text{thnf}} &= \frac{\mu_f}{(1 - \phi_{\text{ZrO}_2} - \phi_{\text{Cu}} - \phi_{\text{Al}_2\text{O}_3})^{2.5}}, \\
\rho_{\text{thnf}} &= (1 - \phi_{\text{ZrO}_2} - \phi_{\text{Cu}} - \phi_{\text{Al}_2\text{O}_3})\rho_f + \phi_{\text{ZrO}_2}\rho_{s1} + \phi_{\text{Cu}}\rho_{s2} + \phi_{\text{Al}_2\text{O}_3}\rho_{s3}, \\
(\rho c_p)_{\text{thnf}} &= (1 - \phi_{\text{ZrO}_2} - \phi_{\text{Cu}} - \phi_{\text{Al}_2\text{O}_3})(\rho c_p)_f + \phi_{\text{ZrO}_2}(\rho c_p)_{s1} + \phi_{\text{Cu}}(\rho c_p)_{s2} \\
&\quad + \phi_{\text{Al}_2\text{O}_3}(\rho c_p)_{s3},
\end{aligned}$$

$$\begin{aligned}
 (\rho\beta)_{\text{thnf}} &= (1 - \phi_{\text{ZrO}_2} - \phi_{\text{Cu}} - \phi_{\text{Al}_2\text{O}_3})(\rho\beta)_f + \phi_{\text{ZrO}_2}(\rho c_p)_{s1} + \phi_{\text{Au}}(\rho c_p)_{s2} \\
 &\quad + \phi_{\text{Al}_2\text{O}_3}(\rho c_p)_{s3}, \\
 \sigma_{\text{thnf}} &= \left[\frac{\sigma_{s1} + 2\sigma_{\text{hnf}} - 2\phi_{\text{ZrO}_2}(\sigma_{\text{hnf}} - \sigma_{s1})}{\sigma_{s1} + 2\sigma_{\text{hnf}} + \phi_{\text{ZrO}_2}(\sigma_{\text{hnf}} - \sigma_{s1})} \right] \sigma_{\text{hnf}}, \sigma_{\text{hnf}} \\
 &= \left[\frac{\sigma_{s2} + 2\sigma_{\text{nf}} - 2\phi_{\text{Cu}}(\sigma_{\text{nf}} - \sigma_{s2})}{\sigma_{s2} + 2\sigma_{\text{nf}} + \phi_{\text{Cu}}(\sigma_{\text{nf}} - \sigma_{s2})} \right] \sigma_{\text{nf}}, \\
 \sigma_{\text{nf}} &= \left[\frac{\sigma_{s3} + 2\sigma_f - 2\phi_{\text{Al}_2\text{O}_3}(\sigma_f - \sigma_{s3})}{\sigma_{s3} + 2\sigma_f + \phi_{\text{Al}_2\text{O}_3}(\sigma_f - \sigma_{s3})} \right] \sigma_f, \\
 k_{\text{thnf}} &= \left[\frac{k_{s2} + 2k_{\text{nf}} - 2\phi_{\text{ZrO}_2}(k_{\text{nf}} - k_{s2})}{k_{s2} + 2k_{\text{nf}} + \phi_{\text{ZrO}_2}(k_{\text{nf}} - k_{s2})} \right] k_{\text{hnf}} k_{\text{hnf}} \\
 &= \left[\frac{k_{s1} + 2k_{\text{hnf}} - 2\phi_{\text{Cu}}(k_{\text{hnf}} - k_{s1})}{k_{s1} + 2k_{\text{hnf}} + \phi_{\text{Cu}}(k_{\text{hnf}} - k_{s1})} \right] k_{\text{nf}} \\
 k_{\text{nf}} &= \left[\frac{k_{s3} + 2k_f - 2\phi_{\text{Al}_2\text{O}_3}(k_f - k_{s3})}{k_{s3} + 2k_f + \phi_{\text{Al}_2\text{O}_3}(k_f - k_{s3})} \right] k_f.
 \end{aligned}$$

2.6. Notations and physical parameters

Tables 2 and 3 present the nomenclature of the various symbols used throughout the formulation of the governing equations.²³

2.7. Dimensionless variable transformation

To facilitate the analysis, we introduce the following local similarity transformations and dimensionless variables:

$$\chi = \frac{\tilde{y}}{h}, \quad \tilde{u} = m\tilde{x}p'(\chi), \quad \tilde{v} = -mhp(\chi), \quad w = mxq(\chi), \quad \vartheta(\chi) = \frac{T - T_0}{T_a - T_0}. \quad (12)$$

The transformation is applied to solve the momentum and thermal behavior of the CMC–water-based base fluid and nanoparticles ZrO₂, Cu and Al₂O₃.

Table 2. Nomenclature of symbols used in the governing equations.

Symbol	Description	Symbol	Description
$\tilde{u}, \tilde{v}, \tilde{w}$	Velocity components in $\tilde{x}, \tilde{y}, \tilde{z}$ directions	$\tilde{x}, \tilde{y}$	Spatial coordinates in horizontal and vertical axes
ρ_{thnf}	Density of the nanofluid	μ_{thnf}	Dynamic viscosity of the nanofluid
κ_{thnf}	Thermal conductivity of the nanofluid	σ_{thnf}	Electrical conductivity of the nanofluid
$(\rho C_p)_{\text{thnf}}$	Heat capacity of the nanofluid	β	Casson fluid parameter
B_0	Magnetic field strength	Ω	Angular velocity of rotating frame
p^*	Modified pressure	q_r	Radiative heat flux
Q_0	Heat generation/absorption coefficient	T	Fluid temperature
T_a	Temperature at the lower wall	T_0	Temperature at the upper wall ($T_a > T_0$)
$U_w = m\tilde{x}$	Stretching velocity at lower wall	v_0	Suction/injection velocity at upper wall
h	Channel height	m	Stretching rate constant

Table 3. Symbols and description.

Symbol	Description
$E_{\text{cm}} = \frac{U_w^2 \rho_f}{(T_a - T_0)(\rho C_P)_f}$	Modified Eckert number
$Q = \frac{Q_0 h^2}{h}$	Heat source parameter
$\delta = \frac{\rho_f U_w^2}{h}$	Thermal relaxation parameter
$N_R = \frac{16}{3} \frac{\sigma^*}{\kappa_f} \frac{T_a^3}{\kappa_f}$	Solar thermal radiation parameter
$B_3 = \frac{\Omega h^2}{\nu_f}$	Rotation parameter
$\text{Ec} = \frac{m^2 h^2 \rho_f}{(T_a - T_0)(\rho C_P)_f}$	Eckert number
$B_7 = \frac{(\rho C_P)_{\text{thnf}}}{(\rho C_P)_f}, B_5 = \frac{\sigma_{\text{thnf}}}{\sigma_f}, B_6 = \frac{\kappa_{\text{thnf}}}{\kappa_f}$	Nanoparticle volume fraction
$B_1 = \frac{\rho_f}{\rho_f}, B_2 = \frac{\mu_{\text{thnf}}}{\mu_f}$	
$\text{Pr} = \frac{(\rho C_P)_f \mu_f}{\rho_f \kappa_f}$	Prandtl number
$B_4 = \frac{m h^2}{\nu_f}$	Dimensionless Reynolds number
$M = \frac{\sigma_f B_0^2 h^2}{\rho_f \nu_f}$	Magnetic parameter
$\nu_f = \frac{\mu_f}{\rho_f}$	Kinematic viscosity

$$\left(1 + \frac{1}{\beta}\right) \frac{B_2}{B_1} p^{(iv)} - B_4(p'p'' - pp''') - 2B_3q' - \frac{B_5}{B_1} Mp'' = 0, \quad (13)$$

$$\left(1 + \frac{1}{\beta}\right) \frac{B_2}{B_1} q'' - B_4(qp' - pq') + 2B_3p' - \frac{B_5}{B_1} Mg = 0, \quad (14)$$

$$\begin{aligned} &\left(1 + \frac{N_R}{B_6}\right) \vartheta'' + \frac{B_7 B_4}{A_6} \text{Pr} p \vartheta' + \frac{Q}{B_6} \vartheta + \frac{B_5}{B_6} \text{Pr} M E_{\text{cm}} (p'^2 + q'^2) \\ &+ \frac{B_2}{B_6} \text{Pr} \left(1 + \frac{1}{\beta}\right) [E_c(4p' + q'^2) + E_{\text{cm}}(p''^2 + q''^2)] + B_6 \delta (f \vartheta'' + p' \vartheta') = 0. \end{aligned} \quad (15)$$

Boundary Conditions:

$$\begin{aligned} p(0) &= 0, & p'(0) &= 1, & q(0) &= 0, & \vartheta(0) &= 1, & \text{at } \chi &= 0, \\ p(1) &= S, & p'(1) &= 0, & q(1) &= 0, & \vartheta(1) &= 0, & \text{at } \chi &= 1. \end{aligned} \quad (16)$$

2.8. Numerical solution

Chebyshev collocation method

To solve the nonlinear coupled ODEs (13)–(16) resulting from the similarity transformation of the 3D boundary layer equations with Cattaneo–Christov heat flux, we apply the Chebyshev collocation method (CCM). This spectral technique is also characterized by its superior performance in finding solutions to problems governed by boundary value by imposing the residuals of the governing equations to be equal to zero at Chebyshev–Gauss–Lobatto points.

Step 1. Domain mapping

We map the physical domain $\chi \in [0, \chi_\infty]$ to the Chebyshev domain $\xi \in [0, 1]$ via

$$\chi = \frac{\chi_\infty}{2}(1 + \xi). \quad (17)$$

Step 2. Collocation points

$$\xi_j = \cos\left(\frac{j\pi}{N}\right), j = 0, 1, \dots, N. \quad (18)$$

Then transform to the physical domain:

Step 3. Approximation using Chebyshev series

$$\chi = \frac{\chi_\infty}{2}(1 + \xi_j). \quad (19)$$

The unknown functions $p(\chi), q(\chi), \vartheta(\chi)$ are approximated by finite Chebyshev series of the first kind:

$$p(\chi) \approx \sum_{k=0}^N a_k T_k(\xi), \quad q(\chi) \approx \sum_{k=0}^N b_k T_k(\xi), \quad \vartheta(\chi) \approx \sum_{k=0}^N c_k T_k(\xi). \quad (20)$$

Derivatives are calculated via Chebyshev differentiation matrices:

$$p' \approx Da, \quad p'' \approx D^2 a, \quad p''' \approx D^3 a, \quad p^{(iv)} \approx D^4 a, \quad (21)$$

$$q' \approx Db, \quad q'' \approx D^2 b, \quad \vartheta' \approx Dc, \quad \vartheta'' \approx D^2 c. \quad (22)$$

Step 4. Substituting the series into the governing equations

At each collocation point, substitute the series into the ODEs to form the residuals.

$$\begin{aligned} R_1^{(j)} = & \left(1 + \frac{1}{\beta}\right) \frac{B_2}{B_1} \sum_{k=0}^N a_k T_k^{(iv)}(\xi_j) \\ & - A_3 \left[\left(\sum_{k=0}^N a_k T_k'(\xi_j) \right) \left(\sum_{m=0}^N a_m T_m''(\xi_j) \right) - \left(\sum_{k=0}^N a_k T_k(\xi_j) \right) \left(\sum_{m=0}^N a_m T_m^{(3)}(\xi_j) \right) \right] \\ & - 2A_4 \sum_{k=0}^N b_k T_k'(\xi_j) - \frac{B_5}{B_1} M \sum_{k=0}^N a_k T_k''(\xi_j), \end{aligned} \quad (23)$$

$$\begin{aligned} R_1^{(j)} = & \left(1 + \frac{1}{\beta}\right) \frac{B_2}{B_1} \sum_{k=0}^N a_k T_k^{(iv)}(\xi_j) - A_3 \left[\left(\sum_{k=0}^N a_k T_k'(\xi_j) \right) \left(\sum_{m=0}^N a_m T_m''(\xi_j) \right) \right. \\ & \left. - \left(\sum_{k=0}^N a_k T_k(\xi_j) \right) \left(\sum_{m=0}^N a_m T_m^{(3)}(\xi_j) \right) \right] - 2A_4 \sum_{k=0}^N b_k T_k'(\xi_j) \\ & - \frac{B_5}{B_1} M \sum_{k=0}^N a_k T_k''(\xi_j), \end{aligned} \quad (24)$$

$$\begin{aligned}
 R_3^{(j)} = & \left(1 + \frac{N_R}{B_6}\right) \sum_{k=0}^N c_k T_{K''}(\xi_j) + \frac{B_3 B_7}{B_6} \Pr \left(\sum_{k=0}^N a_k T_k(\xi_j) \right) \left(\sum_{m=0}^N c_m T_{m'}(\xi_j) \right) \\
 & + \frac{Q}{B_6} \sum_{k=0}^N c_k T_k(\xi_j) + \frac{B_5}{B_6} \Pr ME_{\text{cm}} \left[\left(\sum_{k=0}^N a_k T_{k'}(\xi_j) \right)^2 + \left(\sum_{k=0}^N b_k T_k(\xi_j) \right)^2 \right] \\
 & + \frac{B_2}{B_6} \Pr \left(1 + \frac{1}{\beta} \right) \left[E_c \left(4 \left(\sum_{k=0}^N a_k T_{k'}(\xi_j) \right)^2 + \left(\sum_{k=0}^N b_k T_k(\xi_j) \right)^2 \right) \right. \\
 & \left. + E_{\text{cm}} \left(\left(\sum_{k=0}^N a_k T_{k''}(\xi_j) \right)^2 + \left(\sum_{k=0}^N b_k T_{k'}(\xi_j) \right)^2 \right) \right] + B_6 \delta \\
 & \times \left[\left(\sum_{k=0}^N a_k T_k(\xi_j) \right) \left(\sum_{m=0}^N c_m T_{m''}(\xi_j) \right) + \left(\sum_{k=0}^N a_k T_{k'}(\xi_j) \right) \right. \\
 & \left. \times \left(\sum_{m=0}^N c_m T_{m'}(\xi_j) \right)^1 \right]. \tag{25}
 \end{aligned}$$

Step 5. Imposing boundary conditions

We overwrite specific rows of the residual system to enforce boundary conditions. Suppose we denote the index $J = 0$ for $\chi = 0$ and $j = N$ for χ_∞ . Then

$$\begin{aligned}
 p(0) = 0 & \Rightarrow \sum_{k=0}^N a_k = 0, \quad p'(0) = 0 \Rightarrow (Da)_0 = 0, \\
 q(0) = 0 & \Rightarrow \sum_{k=0}^N b_k = 0, \quad \vartheta(0) = 1 \Rightarrow \sum_{k=0}^N c_k = 1, \\
 p'(\chi_\infty) = 0 & \Rightarrow (Da)_N = 0, \\
 q(\chi_\infty) = 0 & \Rightarrow \sum_{k=0}^N (-1)^k b_k = 0, \\
 \vartheta(\chi_\infty) = 0 & \Rightarrow \sum_{k=0}^N (-1)^k c_k = 0.
 \end{aligned} \tag{26}$$

Step 7. Solve the nonlinear system assemble of the full residual vector, where

$$U = [a_0, a_1, \dots, a_N, b_0, b_1, \dots, b_N, C_0, \dots, C_N]^T. \tag{27}$$

Solve using the Newton–Raphson method:

$$U_{k+1} = U_k - J^{-1}(U_k) \cdot R(U_k). \tag{28}$$

Table 4 shows the convergence for skin friction coefficients, Nusselt number and CPU time. As N (the amount of Chebyshev collocation point) increases, the values of the skin friction coefficients and Nusselt number stabilize. This proves the fact that the Chebyshev spectral method has spectral convergence. The comparison of the Nusselt number, as presented in Table 5, demonstrates close agreement between the current results and existing literature.

Table 4. Convergence, residual error and CPU time for skin friction coefficients and Nusselt number.

Collocation points N	C_{px}	Residual error in C_{px}	C_{qz}	Residual error in C_{qz}	Nu	Residual error in Nu	CPU time (3)
5	0.7321	—	0.5143	—	0.9065	—	0.008
10	0.7325	0.0004	0.5146	0.0003	0.9069	0.0004	0.017
15	0.7326	0.0001	0.5147	0.0001	0.9070	0.0001	0.036
20	0.7326	0.00001	0.5147	0.00001	0.9070	0.00001	0.070
25	0.7326	0.000001	0.5147	0.000001	0.9070	0.000001	0.133
30	0.7326	0.0000001	0.5147	0.0000001	0.9070	0.0000001	0.220

Table 5. Comparison of the Nusselt number with existing literature.

ϕ	Current	Outcome of 38	ϕ	Current	Outcome of 39
0.0	1.40365858	1.40437	0.0	2.19499683	2.19498
0.05	1.61905279	1.61931	0.1	1.40500022	1.40500
0.1	1.88947420	1.88889	0.2	1.20404685	1.20405

3. Results and Discussion

The thermal performance of PVT cooling systems was numerically analyzed using a non-Fourier heat conduction framework, considering the combined effects of solar thermal radiation and rotational flow. Parametric studies were undertaken to explore the influence of governing parameters. The numerical simulations were carried out by assigning appropriate default values and parametric ranges as follows: nanoparticles volume fraction ($\phi = 0.05, 0.15, 0.25$), Casson fluid parameter ($\beta = 0.2, 0.3, 0.4$), Reynolds number ($B_4 = 0.1, 0.2, 0.3$), rotation parameter ($B_3 = 0.4, 0.5, 0.6$), magnetic parameter ($M = 0.5, 0.6, 0.7$), solar thermal radiation parameter ($R = 0.1, 0.2, 0.3$), heat generation ($Q = 0.2, 0.3, 0.4$), Eckert number ($Ec = 0.1, 0.5, 0.7$) and the modified Eckert number ($E_{cm} = 0.1, 0.5, 0.7$). Table 6 demonstrates how variations in physical parameters: the magnetic parameter, nanoparticles volume fraction, Eckert number and heat source/sink parameter, influence the skin friction coefficient and Nusselt number. These parameters are vital in governing thermal and velocity transport of PVT cooling systems. The rising value of M suppresses the skin friction coefficient and boosts the Nusselt number due to magnetic damping and Joule heating. Increasing ϕ increases thermal performance but at the cost of reduced flow speed. A greater value of Ec upsurges the Nusselt number through internal viscous heating. The rising value of the heat generation parameter leads to an increased Nusselt number. Also, the skin friction coefficient drops with rising value β .

Nanoparticles volume fraction (ϕ): The influence of nanoparticle volume fraction on fluid flow and heat transfer characteristics, as shown in Figs. 2(a) and 2(b), reveals a coupled behavior that is highly relevant to the performance of PVT cooling systems. In Fig. 2(a), it is evident that increasing the nanoparticle volume fraction

Table 6. Effect of physical parameters on skin friction coefficient and Nusselt number.

ϕ	$-\phi' (0)$	$-\theta' (1)$	$f'' (0)$	$f'' (1)$
0.05	-9.9500	8.4500	2.0000	-1.3500
0.15	-9.7000	8.4500	2.0450	-1.3500
0.25	-9.4500	8.4500	2.0900	-1.3500
β	$-\theta' (0)$	$-\theta' (1)$	$f'' (0)$	$f'' (1)$
1	-9.7000	6.4500	2.0450	-1.7505
3	-9.7000	8.4500	2.0450	-1.2282
5	-9.7000	10.4500	2.0450	-0.9501
M	$-\theta' (0)$	$-\theta' (1)$	$f'' (0)$	$f'' (1)$
0.5	-9.9500	8.4500	2.0000	-1.3500
1.5	-9.7000	8.4500	2.0450	-1.3500
2.5	-9.4500	8.4500	2.0900	-1.3500
Ec	$\theta' (0)$	$-\theta' (1)$	$f'' (0)$	$f'' (1)$
0.1	-9.7000	8.1500	2.0992	-1.4500
0.3	-9.7000	8.4500	2.0992	-1.3432
0.5	-9.7000	8.7500	2.0992	-1.2500
S	$-\theta' (0)$	$-\theta' (1)$	$f'' (0)$	$f'' (1)$
0	-9.3202	8.3511	2.0430	-1.3541
1	9.3202	8.4419	2.1051	-1.4135
2	9.3202	8.5124	2.1434	-1.5953

leads to a reduction in velocity distribution. This occurs because the introduction of nanoparticles increases both the effective viscosity and density of the nanofluid, which in turn enhances internal resistance to motion and raises the inertia of the fluid. As more particles are suspended in the base fluid, momentum transfer between

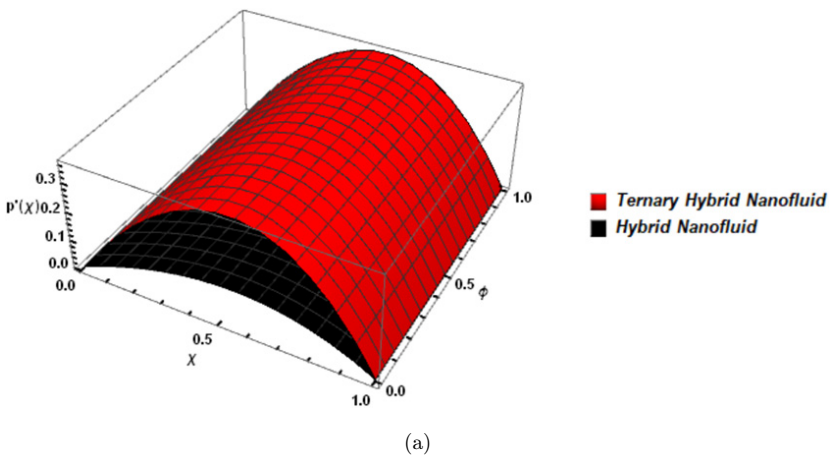
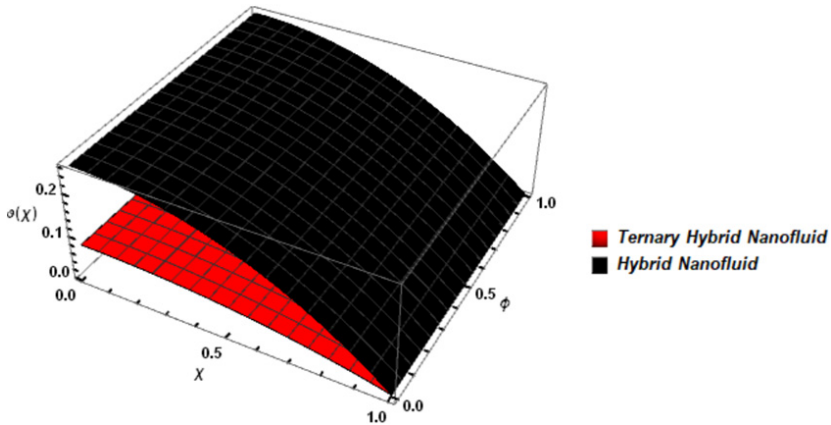
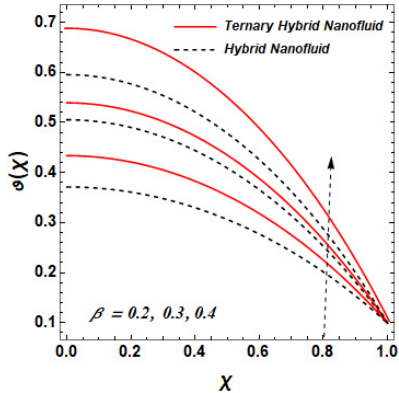


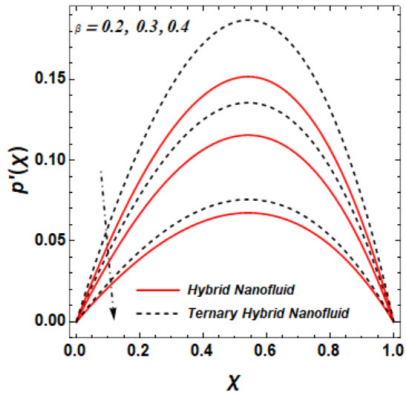
Fig. 2. (Color online) (a) 3D plot of ϕ on $p'(\chi)$. (b) 3D plot of ϕ on $\vartheta(\chi)$. (c) Impact β on $\vartheta(\chi)$. (d) Impact β on $p'(\chi)$. (e) Impact B_4 on $\vartheta(\chi)$.



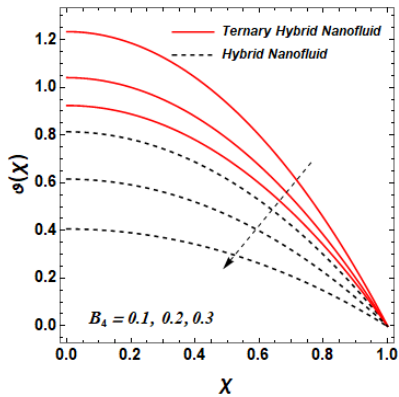
(b)



(c)



(d)



(e)

Fig. 2. (Continued)

fluid molecules and nanoparticles becomes stronger, leading to greater drag forces that suppress the overall flow rate. Consequently, the velocity profile diminishes as nanoparticle concentration rises. Figure 2(b) demonstrates that the same increase in nanoparticle volume fraction results in an enhancement of the temperature distribution. This outcome is directly tied to the superior thermal conductivity of nanoparticles compared with the base fluid. As nanoparticles are added, the ability of the nanofluid to absorb, conduct and distribute heat is improved, enabling more efficient heat transfer within the PVT cooling channel.

Casson fluid parameter (β): The effect of the Casson fluid parameter on temperature and velocity distribution provides deep insight into the behavior of non-Newtonian fluids in complex thermal and flow systems. As illustrated in Fig. 2(c), an increase in the Casson fluid parameter enhances the temperature distribution, while Fig. 2(d) reveals that the velocity profile is suppressed with the same parameter variation. Physically, this behavior can be attributed to the unique characteristics of Casson fluids, which deviate significantly from Newtonian fluids due to the presence of yield stress. Unlike Newtonian fluids that flow freely under shear, Casson fluids resist motion until a critical shear stress threshold is exceeded. When the Casson parameter is high, the effective yield stress increases, causing more resistance to fluid deformation. This resistance reduces the flow rate and consequently diminishes the velocity field across the channel. The suppression of velocity, in turn, leads to reduced convective transport of energy, thereby allowing the thermal boundary layer to grow thicker, which manifests as an elevation in temperature within the fluid domain. Thus, the increase in temperature and the simultaneous reduction in velocity can be understood as coupled effects of enhanced viscous resistance and reduced energy convection due to stronger non-Newtonian behavior.

Reynolds number (B_4): The influence of the Reynolds number on thermal distribution is a fundamental aspect of fluid dynamics and heat transfer analysis, particularly in systems involving forced convection and energy transport. As illustrated in Fig. 2(e), an increase in the Reynolds number leads to a noticeable reduction in the thermal distribution within the flow domain. Physically, the Reynolds number represents the ratio of inertial forces to viscous forces in the fluid, and its variation determines whether the flow is dominated by diffusion or convection. At higher Reynolds numbers, inertial forces become stronger, resulting in an enhanced convective transport of energy. This enhanced convection accelerates the movement of fluid particles, allowing heat to be carried away more effectively from heated regions. Consequently, the thermal boundary layer becomes thinner, and the overall temperature distribution across the domain reduces because heat is dissipated more rapidly into the bulk of the fluid. In essence, a higher Reynolds number corresponds to stronger convective cooling and weaker thermal buildup, whereas lower Reynolds numbers support thicker boundary layers and greater thermal accumulation.

Rotation parameter (B_3): The effect of the rotation parameter on thermal distribution offers significant insights into the thermomechanical behavior of rotating fluid systems. As shown in Fig. 3(a), an increase in the rotation parameter leads to a reduction in the thermal distribution across the fluid domain. Physically, the rotation parameter represents the influence of Coriolis forces on the flow system. When rotation is introduced, additional inertial forces act perpendicular to the fluid's motion, which alters the velocity gradients and redistributes energy transport mechanisms. As the rotation parameter increases, the fluid particles experience stronger rotational forces, which induce secondary circulations and modify the boundary layer structure. This enhanced rotational influence tends to thin

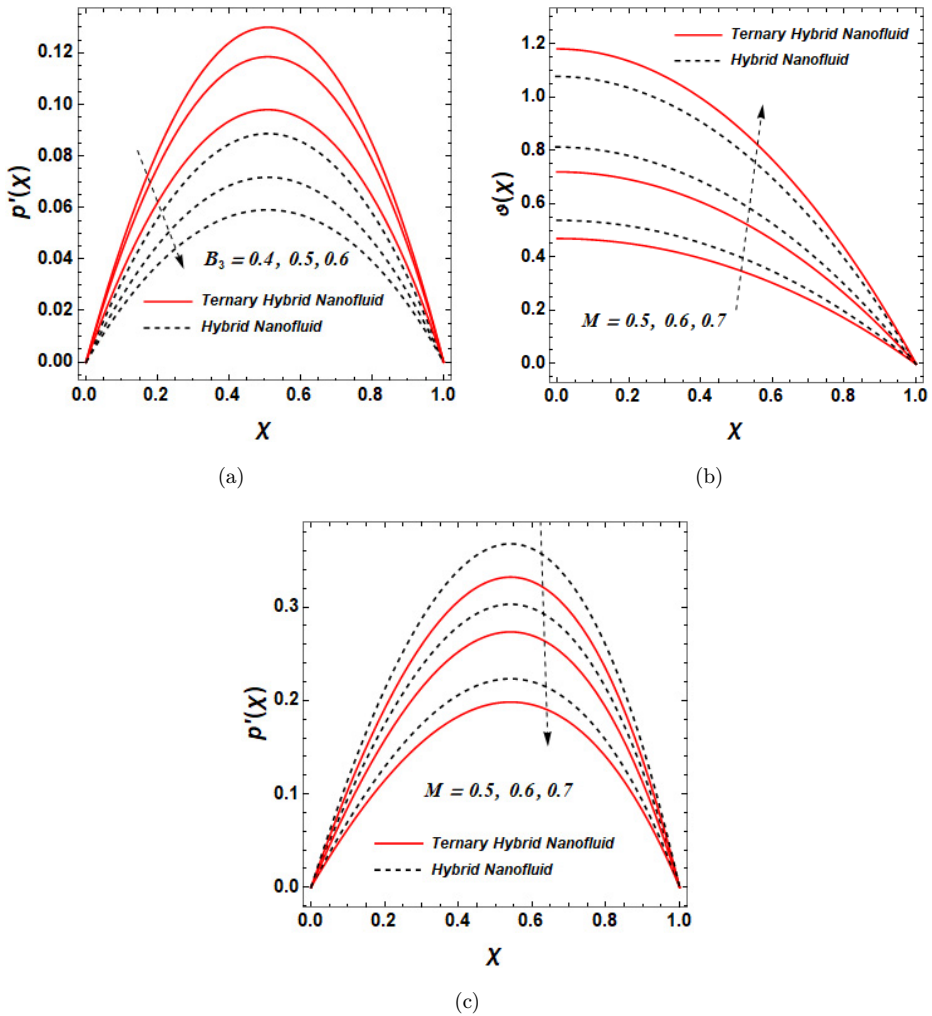


Fig. 3. (Color online) (a) Impact B_3 on $\vartheta(X)$. (b) Impact M on $\vartheta(X)$. (c) Impact M on $p'(X)$.

the thermal boundary layer, as the convective transport of heat is redirected and intensified by the swirling motion of the flow. Consequently, the elevated mixing induced by rotation accelerates the dissipation of heat from the heated surface into the bulk of the fluid, thereby reducing the overall thermal distribution within the system.

The magnetic parameter (M) plays a critical role in governing the thermal and hydrodynamic characteristics of electrically conducting nanofluid flows, especially within systems where MHD effects are important. In the present analysis, the influence of the magnetic parameter on both temperature and velocity distributions is illustrated in Figs. 3(c) and 3(d), respectively. It is observed that an increase in the magnetic parameter leads to a pronounced enhancement in the fluid temperature profile (Fig. 3(c)), while simultaneously producing a reduction in the velocity distribution (Fig. 3(d)). This dual effect is rooted in the fundamental interaction between the induced Lorentz force and the fluid flow characteristics. When an external magnetic field is applied, the movement of charged particles within the conducting nanofluid gives rise to a resistive Lorentz force. This force acts opposite to the direction of fluid motion, thereby imposing additional drag that suppresses velocity. As a result, the flow decelerates significantly with increasing M , as demonstrated in Fig. 3 (d). The retardation of flow reduces convective heat transport, which in turn allows greater thermal energy to be retained within the system. Consequently, the temperature distribution rises noticeably with higher magnetic interaction, as illustrated in Fig. 3(c). This effect highlights the intrinsic thermal–momentum coupling in MHD flows, where momentum suppression directly enhances heat accumulation.

Solar thermal radiation parameter (R): The influence of the solar thermal radiation parameter on thermal distribution, as displayed in Fig. 4(a), reveals a clear

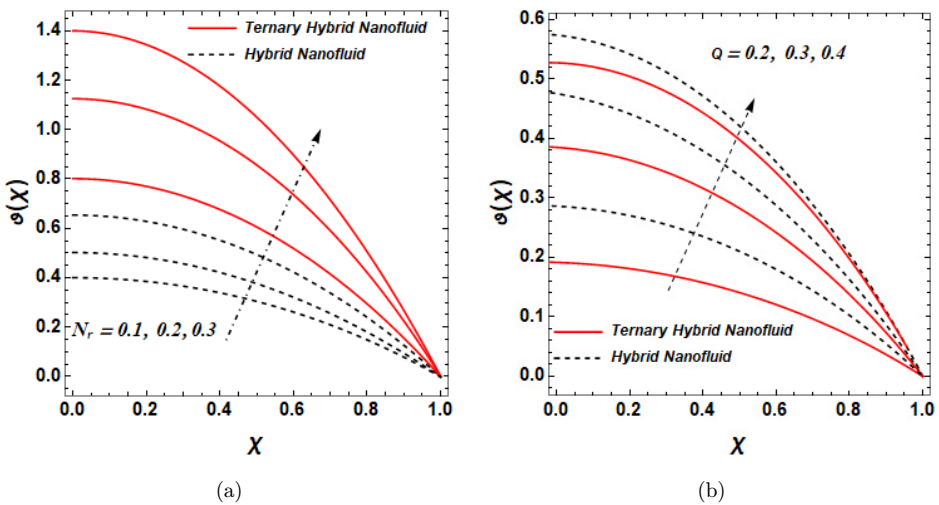


Fig. 4. (Color online) (a) Impact N_r on $\vartheta(\chi)$. (b) Impact Q on $\vartheta(\chi)$.

trend where temperature intensifies with increasing radiation effects. This occurs because solar radiation serves as a direct external energy source, introducing additional thermal energy into the system. As the radiation parameter increases, the absorption of solar energy by the fluid medium intensifies, which in turn raises the energy levels of fluid molecules and enhances the temperature distribution across the domain. In essence, solar photons act as carriers of energy that, upon interaction with the medium, elevate the internal energy and promote stronger molecular agitation, thereby increasing the thermal field in a nonlinear manner. This interaction is particularly prominent in models where radiative–conductive heat transfer is considered, as radiation contributes comparably or even dominantly to overall heat transfer relative to conduction and convection.

Heat generation (Q): Figure 4(b) displays that an increase in Q leads to a corresponding rise in $\vartheta(\chi)$ throughout the fluid medium. This outcome is a direct consequence of internal volumetric heat sources embedded within the fluid system, which continuously supply thermal energy to the fluid. The heat generation parameter quantifies the intensity of this internal heat source, and as it upsurges, it raises the energy content in the fluid, thereby raising the $\vartheta(\chi)$.

Eckert number (Ec) and the modified Eckert number (E_{cm}): The impact of the Eckert number and the modified Eckert number on thermal distribution, as shown in Figs. 5(a) and 5(b), indicate a remarkable increase in the $\vartheta(\chi)$ when these parameters are improved. The Eckert number (Ec) is a dimensionless parameter giving the kinetic energy in relation to the enthalpy difference and determines how the kinetic energy is transformed to internal energy by means of viscous dissipation. In the case of fluid flow at significant velocity gradients, it gives frictional heating, especially in the boundary layer and near-wall areas. This is an increasing effect of

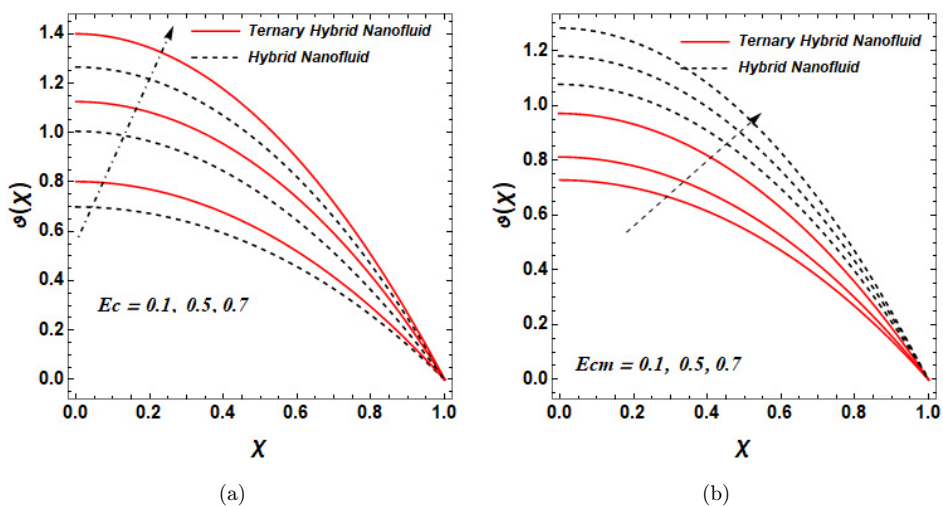


Fig. 5. (Color online) (a) Impact Ec on $\vartheta(\chi)$. (b) Impact E_{cm} on $\vartheta(\chi)$.

the Eckert number, which shows that more kinetic energy of the fluid is being converted into heat. The higher the Eckert number is, the more the mechanical energy of the fluid is used to transform into internal heat energy via viscous dissipation. This conversion introduces thermal energy directly into the system, thus raising the fluid heat in the boundary layer. In physical terms, this increment of temperature distribution can be explained by the increased mechanisms of energy dissipation in the flow. The nanoparticle-based fluid interactions play a role in the determination of the thermal conductivity and internal energy storage capacity of a system undergoing nanofluids or MHD flows.

4. Conclusions

In conclusion, this study investigated enhanced thermal regulation in PVT cooling systems using the Cattaneo–Christov heat flux model under the influence of solar radiation and rotating MHD flow. The findings demonstrate that incorporating non-Newtonian nanofluids significantly improves both thermal and flow performance in PVT systems. By employing nanofluids within a Casson fluid framework, the work provides valuable insights into advanced strategies for effective thermal management. The key outcomes are summarized as follows:

- As the β increases, thermal conductivity increases due to decreased fluid resistance, making it ideal for thermal management in biomedical and industrial flows.
- Casson liquids display great yield stress, which suppresses motion, particularly in thicker biofluids, aiding flow control in microchannels.
- Greater Re improves convective cooling and decreases temperature buildup, significant for high-speed flows.
- A_3 parameter induces centrifugal effects that improve thermal diffusion, appropriate for rotating machinery.
- Adding more nanoparticles increases thermal conductivity, improving thermal transport, ideal for nanofluid cooling technologies.
- Magnetic fields induce Joule heating, increasing thermal profile, applicable in magnetic drug targeting and MHD systems.
- Solar radiation improves thermal energy systems, rising the temperature beneficial in solar collectors and thermal energy storage.
- As kinetic energy converts to heat, the thermal profile increases, relevant for compressible flows and heating in mechanical processes.

ORCID

Leila Manai  <https://orcid.org/0000-0003-4369-3006>

M. J. Johnson  <https://orcid.org/0009-0008-6214-4495>

Saleh Chebaane  <https://orcid.org/0000-0001-6885-8634>

A. M. Obalalu  <https://orcid.org/0000-0002-7597-0525>

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