

STUDY OF THE ALPHA DECAY HALF-LIVES FOR THORIUM ISOTOPES IN THE MASS RANGE 212 – 232

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ABSTRACT

Theoretical study of the α -decay half-lives of Thorium isotopes ($Z = 90$) in the mass range $212 \leq A \leq 232$ was carried out using three phenomenological models viz. the Gamow-like model (GLM), the modified Gamow-like model (MGLM), and the proximity potential model. The temperature dependence proximity interaction is also considered. The results are compared with experimental data and other theoretical calculations. The calculated standard deviation (σ) indicates that the modified Gamow-like model (using the newly calculated parameters), with $\sigma = 0.3245$, gives the lowest deviation from experimental values. This suggests that the MGLM is the most suitable model for describing the alpha-decay half-lives of the Thorium isotopes. The MGLM model gives more results than the models used in the literature.

Keywords: Alpha decay, Proximity potential, Gamow-like model.

INTRODUCTION

Alpha decay, first discovered in 1899 by Rutherford, is one of the important decay modes of radioactive nuclei (Shin et al., 2016; Zdeb et al., 2013). The decay mode has successfully been described using quantum theory (Hassanabadi et al., 2013). It provides nuclear structure information, identification and stability of heavy and superheavy nuclei (SHN) (Alsaif et al., 2017; Cheng et al., 2019; Santhosh et al., 2020). Various theoretical investigation of α -decay half-lives have been carried out for heavy and superheavy nuclei. Different theoretical models such as the Cluster formation model (Ahmed, 2017; Ahmed et al., 2013; Deng and Ren, 2016; Deng et al., 2015, 2018), the generalized liquid drop model (Royer and Moustabchir, 2001; Royer and Zhang, 2008; Xiaojun et al., 2014), the modified generalized liquid drop model (Santhosh et al., 2020; Santhosh and Jose, 2019; Santhosh et al., 2018), the effective liquid drop model (Cui et al., 2018), the fission-like model (Wang et al., 2010), and the preformed cluster model (Gupta and Greiner, 1994; Singh et al., 2010), have been employed to carry out the α -decay half-life studies. These models use various interactions ranging from the phenomenological potential such as the Woods Saxon, proximity potentials etc to microscopic interactions like the deformed density dependent potential. Some analytical formulas have also been introduced to compute the α -decay half-lives of many isotopes. Some of these formulas are the universal decay law developed by Qi et al. (2009a,b), the Royer formula (Royer, 2000, 2010; Royer et al., 2011),

Viola-Seaborg formula (Viola and Seaborg, 1966), the Akrawy formula (Akrawy and Ahmed, 2018), the Ren formula, (Akrawy et al., 2019; Ren et al., 2004), the scaling law of Horoi (Horoi, 2004), scaling law of Brown, the AKRE formula developed by Akrawy and Poenaru (Akrawy and Poenaru, 2017) among others.

Zdeb et al. (2013) introduced a model, based on the Gamow theory, to compute the α -decay half-lives of heavy nuclei with $Z \geq 84$. In the Gamow-like model, the author employs the square well as the nuclear potential, and the Coulomb potential is taken to be a uniformly charged sphere (Zdeb et al., 2013). Following a similar approach, Cheng et al. (2019) used an improved interaction by using a modified Coulomb interaction and taking into consideration the centrifugal interaction. These potential models are used to calculate the α -decay half-lives of the Th isotopes in this study. The three adjustable parameters in the Gamow-like model viz. the radius constant r_0 , the hindrance factor h (and the screening parameter a of the modified interaction) are calculated for the $^{212-232}\text{Th}$ isotopes. The parameters are fully described in the following Section. We have also used the Coulomb and proximity potential model to calculate the half-lives. Both the temperature-independent and temperature-dependent versions are used with the prox. 2010 potential. The results of the calculations are compared with experimental data and other theoretical results. The theoretical calculations of some even-even Thorium isotopes has already been carried out using the double folding model with the effective nucleon-nucleon

potential Yukawa (M3Y)-Ried-type interaction (Ghorbani et al., 2020), and the proximity potential model with the prox. 1987 potential. Good agreement with experimental data was obtained using the M3Y potential, while the proximity potential was not successful. The article is organized as follows. In Section 2, the theoretical models for the calculation of the α -

decay half-lives are presented; the Gamow-like model, modified Gamow-like model, temperature-independent Coulomb and proximity potential model, and temperature-dependent Coulomb and proximity potential model. The results of the calculations are presented and discussed in Section 3 while the conclusion is given in Section 4.

THEORY

Gamow-like model (GLM)

In the Gamow-like model, the α -decay half-life is calculated using (Zdeb et al., 2013)

$$T_{1/2} = \frac{\ln 2}{\lambda} 10^h, \quad (1)$$

where h is the decay hindrance factor which is due to the effect of an odd-neutron and/or an odd-proton. The value is zero for even-even nuclei. For odd-nucleon $h = h_p = h_n = 0.216$, $h_{np} = 2h$, as determined by Zdeb et al. (2013). The decay constant λ is computed using (Zdeb et al., 2013):

$$\lambda = \nu P, \quad (2)$$

where the assault frequency is given as (Zdeb et al., 2013):

$$\nu = \frac{\pi h}{2\mu R^2}, \quad (3)$$

and the α -particle probability (P) of tunneling through the barrier is given by (Zdeb et al., 2013):

$$P = \exp \left[-\frac{2}{h} \int_R^b \sqrt{2\mu(V(r) - E_k)} dr \right] \quad (4)$$

Here, $\mu = M_p A_1 A_2 / (A_1 + A_2)$ denotes the reduced mass of the daughter nucleus and the α particle, $M_p = 931.494 \text{ MeV}/c^2$, and E_k denotes the kinetic energy of the emitted α particle. The radius of the spherical square well is calculated by summing the radii of both the daughter nucleus (A_2) and the α particle (A_1) using (Zdeb et al., 2013):

$$R = r_0 \left(A_1^{1/3} + A_2^{1/3} \right) \text{ (fm)}, \quad (5)$$

where r_0 is a constant, an adjustable parameter. It was determined to be 1.21 fm by Zdeb et al. (2013). The point of exit from the barrier b is the point at which the kinetic energy (E_k) is equal to the Coulomb potential (Zdeb et al., 2013):

$$b = \frac{Z_1 Z_2 e^2}{E_k} \text{ (fm)}, \quad (6)$$

where Z_1 and Z_2 are the atomic numbers of the α particle and daughter nucleus, respectively.

The potential energy $V(r)$ in the Gamow-like model (GLM) is given by (Zdeb et al., 2013):

$$V(r) = \begin{cases} -V_0 & 0 \leq r \leq R \\ \frac{Z_1 Z_2 e^2}{r} & r > R \end{cases}, \quad (7)$$

where V_0 is taken to be $25 A_1 \text{ (MeV)}$.

Modified Gamow-like model (MGLM)

In the modified Gamow-like model (Cheng et al., 2019), the interaction potential between the alpha particle and daughter nucleus is given by (Cheng et al., 2019):

$$V(r) = \begin{cases} -V_0, & 0 \leq r \leq R \\ V_H(r) + V_\ell(r), & r \geq R, \end{cases} \quad (8)$$

where the Hulthen type of screened electrostatic Coulomb potential (Cheng et al., 2019):

$$V_H = \frac{a Z_1 Z_2 e^2}{e^{ar} - 1}, \quad (9)$$

the centrifugal potential, with Langer modification (Ghorbani et al., 2020),

$$V_\ell(r) = \frac{\left(\ell + \frac{1}{2}\right)^2 \hbar^2}{2\mu r^2}, \quad (10)$$

V_0 is the depth of the square well, ℓ is the orbital angular momentum that the α particle takes away, and a is the screening parameter. The radius of the spherical square well is calculated using equation (5). The α -decay half-life is calculated using equation (1). In this model, the values of the three parameters (a , r_0 , and h) were determined by Cheng et al. (2019) to be:

$$a = 7.8 \times 10^{-4} \text{ fm}^{-1}, r_0 = 1.14 \text{ fm}, h = 0.3455. \quad (11)$$

The penetration probability P is calculated using equation (4), but the classical turning point b is computed using the condition $V(b) = E_k$. The frequency of assault on the potential barrier and the radius of the parent nucleus are computed using (Cheng et al., 2019):

$$\nu = \frac{(G + \frac{3}{2})\hbar}{1.2\pi\mu R_0^2}, \quad (12)$$

and

$$R_0 = 1.28A^{1/3} - 0.76 + 0.8A^{-1/3} \text{ (fm)} \quad (13)$$

respectively. The main quantum number G is calculated using (Cheng et al., 2019):

$$G = \begin{cases} 22 & N > 126 \\ 20 & 82 < N \leq 126, \\ 18 & N \leq 82 \end{cases} \quad (14)$$

where N is the neutron number.

Coulomb and Proximity Potential Model (CPPM)

The total interaction potential between the emitted and the daughter nuclei in the CPPM contains (for both the touching configuration and for separated fragments) the nuclear, the Coulomb and the centrifugal terms (Santhosh et al., 2015):

$$V_T(r) = V_{\text{prox}}(z) + V_C(r) + \frac{\hbar\ell(\ell+1)}{2\mu r^2}, \quad (15)$$

where μ is the reduced mass of the interaction system and ℓ is the angular momentum. The Coulomb potential $V_C(r)$ is defined as (Santhosh et al., 2015):

$$V_C(r) = Z_1 Z_2 e^2 \begin{cases} \frac{1}{r} & \text{for } r \geq R_C \\ \frac{1}{2R_C} \left[3 - \left(\frac{r}{R_C} \right)^2 \right] & \text{for } r \leq R_C \end{cases} \quad (16)$$

Here, Z_1 and Z_2 are, respectively, the charge numbers of the daughter and emitted nuclei. The radial distance $R_C = 1.24(R_1 + R_2)$ fm. The term $V_{\text{prox}}(z)$ denotes the proximity potential and $z = r - C_1 - C_2$ is the distance between the near surfaces of the fragments, where r is the distance between the fragment centres and C_1 and C_2 are the Süssmann central radii of fragments (Zanganah et al., 2020). The presence of the proximity potential causes a reduction in the height of the potential barrier. Shi and Swiatecki (Shi and Swiatecki, 1985) first used the proximity potential in an empirical manner. The proximity potential V_{prox} can be obtained by calculating the strength of the nuclear interactions between the daughter and emitted nuclei (Santhosh et al., 2015):

$$V_{\text{prox}}(z) = 4\pi b\gamma\bar{R}\phi\left(\frac{z}{b}\right) \text{ MeV}, \quad (17)$$

where the term $b\gamma\bar{R}$ depends on the geometry and shape of the two nuclei, b is the diffuseness of the nuclear surface, γ is the nuclear surface energy coefficient, and the mean curvature radius $\bar{R}$ is given as (Santhosh et al., 2015):

$$\bar{R} = \frac{C_1 C_2}{C_1 + C_2}. \quad (18)$$

The Süssmann central radii of fragments C_1 and C_2 are computed using (Santhosh et al., 2015):

$$C_i = R_i \left[1 - \left(\frac{b}{R_i} \right)^2 + \dots \right], \quad (19)$$

where the diffuseness of nuclear surface $b \approx 1$ fm and R_i ($i = 1, 2$) are as defined in equation (13). The universal function $\phi\left(\varepsilon = \frac{z}{b}\right)$ is given in the form (Gharaei and Zanganah, 2016):

$$\phi(\varepsilon) = \begin{cases} -\frac{1}{2}(\varepsilon - 2.54)^2 - 0.0852(\varepsilon - 2.54)^3 & \varepsilon \leq 1.2511 \\ -3.437\exp(-\varepsilon/0.75) & \varepsilon \geq 1.2511 \end{cases} \quad (20)$$

The nuclear surface energy coefficient γ is defined as (Gharaei and Zanganeh, 2016):

$$\gamma = 1.460734 \left[1 - 4 \left(\frac{N-Z}{N+Z} \right)^2 \right] \text{ MeV/fm}^2, \quad (21)$$

where N and Z denote the neutron and proton numbers of the parent nucleus, respectively.

According to the WKB approximation (Berry and Mount, 1972; Gharaei and Zanganeh, 2016; Gurvitz and Kalbermann, 1987), the penetration probability P of the emitted α nucleus through the potential barrier is calculated using:

$$P = \exp \left[-\frac{2}{\hbar} \int_{R_{in}}^{R_{out}} \sqrt{2\mu[V(r) - Q]} dr \right], \quad (22)$$

where the classical turning points R_{in} and R_{out} are determined from (Gharaei and Zanganeh, 2016):

$$V(R_{in}) = V(R_{out}) = Q, \quad (23)$$

and A is the mass number of the parent nucleus. The α -decay half-life is then computed using (Gharaei and Zanganeh, 2016):

$$T_{1/2} = \frac{\ln 2}{\nu P}, \quad (24)$$

where the assault frequency ν has been taken to be 10^{20} s^{-1} .

Temperature-Dependent Proximity Potential (CPPMT)

The temperature-dependent proximity potential is computed by using the temperature-dependent forms of the parameters R , b , and γ . They are given by (Gharaei and Zanganeh, 2016):

$$R_i(T) = R_i(0)[1 + 0.0005T^2] \text{ fm} \quad (i = 1, 2), \quad (25)$$

$$b(T) = b(0)[1 + 0.009T^2], \quad (26)$$

$$\gamma(T) = \gamma(0) \left[1 - \frac{T - T_b}{T_b} \right]^3, \quad (27)$$

where T_b denotes the temperature associated with near Coulomb barrier energies and $b(T=0) = 1$. In this work, an alternative form of the temperature dependent surface energy coefficient in the form $\gamma(T) = \gamma(0)(1 - 0.07T)^2$ (Ghorbani et al., 2020) has been adopted. The temperature T (in MeV) can be expressed as (Gharaei and Zanganeh, 2016):

$$E^* = E_{kin} + Q_{in} = \frac{1}{9}AT^2 - T, \quad (28)$$

where E^* is the excitation energy of the parent nucleus and A is its mass number. The entrance channel Q -value of the system is denoted as Q_{in} .

RESULTS AND DISCUSSIONS

The results of the α -decay half-lives calculations for the 21 Thorium isotopes ($^{212-232}\text{Th}$) are presented in this Section. The calculations have been carried out using the Gamow-like model (GLM), modified Gamow-like model (MGLM), temperature-independent Coulomb and proximity potential model (CPPM), and temperature-dependent Coulomb and proximity potential model. The Gamow-like model using the parameters of Zdeb et al. (2013) is denoted as GLM1, while the Gamow-like model using parameters determined through a least square fit to experimental values is denoted as GLM2. Same thing applies to the modified Gamow-like model. MGLM1 is the model using the parameters of Cheng et al. (2019) and MGLM2 is the model using parameters determined through a least square fit to experimental data. The calculated parameters for the GLM2 are

$$r_0 = 1.2356 \text{ fm}, \quad h = 1.0453, \quad (29)$$

while the calculated parameters for the MGLM2 are

$$r_0 = 1.1527 \text{ fm}, \quad h = 0.7709, \quad a = 4.6967 \times 10^{-4} \text{ fm}^{-1}. \quad (30)$$

The experimental data used in the calculations have been taken from the NUBASE2016 database (Audi et al., 2017; Huang et al., 2017; Wang et al., 2017).

The reaction Q_α value has been computed via (Santhosh et al., 2015):

$$Q_\alpha = \Delta M_P - \Delta M_D - \Delta M_\alpha + k(Z_P^\varepsilon - Z_D^\varepsilon), \quad (31)$$

where ΔM_p , ΔM_α , and ΔM_d denote the mass excesses of the parent nucleus, the alpha particle, and the daughter nucleus, respectively. The term $k(Z_p^2 - Z_d^2)$ denotes the screening effect of atomic electrons (Denisov and Ikezoe, 2005); $k = 8.7 \text{ eV}$, $\epsilon = 2.517$ for $Z \geq 60$, and $k = 13.6 \text{ eV}$, $\epsilon = 2.408$ for $Z < 60$ (Huang et al., 1976). The angular momentum ℓ are determined from the selection rule given by (Akrawy et al., 2019; Denisov et al., 2012; Qi et al., 2015):

$$\ell = \begin{cases} \Delta_j & \text{for even } \Delta_j \text{ and } \pi_d = \pi_p \\ \Delta_j + 1 & \text{for odd } \Delta_j \text{ and } \pi_d = \pi_p \\ \Delta_j & \text{for odd } \Delta_j \text{ and } \pi_d \neq \pi_p \\ \Delta_j + 1 & \text{for even } \Delta_j \text{ and } \pi_d \neq \pi_p \end{cases} \quad (32)$$

where $\Delta_j = |j_p - j_d|$, and j_d , π_d , j_p , π_p are the spin and parity values of the daughter and parent nuclei, respectively.

The calculated alpha-decay half-lives for the ($^{212-232}\text{Th}$) isotopes are shown in Table 1. The first three columns show the mass number (A), the experimental Q_α values, and the experimental α -decay half-lives (Expt.) ($\log[T_{1/2}(s)]$), respectively. The fourth to ninth columns of the Table show the calculated α -decay half-lives

using the GLM1, GLM2, MGLM1, MGLM2, CPPM, and CPPMT, respectively. The tenth column is the theoretical result of Ghorbani et al. (2020) using the double folding model with the effective nucleon-nucleon potential Yukawa (M3Y)-Ried-type interaction. All the models give very good descriptions of the half-lives.

Table 1: Calculated α -decay half-lives, $\log[T_{1/2}(s)]$, of Th isotopes ($Z = 90$) using GLM1, GLM2, MGLM1, MGLM2, CPPM, and CPPMT. The last column is the theoretical result (Theory) of Ghorbani et al. (2020).

A	Q_α	$\log[T_{1/2}(s)]$							
		Expt.	GLM1	GLM2	MGLM1	MGLM2	CPPM	CPPMT	Theory
212	7.9977	-1.4976	-1.5136	-1.8169	-1.6977	-1.7411	-1.8897	-1.6867	-1.883
213	7.8766	-0.8416	-0.9358	-0.4110	-0.9947	-0.6102	-1.5173	-1.3144	
214	7.8669	-1.0600	-1.1402	-1.4448	-1.3282	-1.3690	-1.4994	-1.2966	-1.510
215	7.7043	0.0792	-0.4158	0.1074	-0.2597	0.1270	-0.7345	-0.5249	
216	8.1121	-1.5850	-1.9427	-2.2457	-2.1191	-2.1652	-2.3005	-2.0980	-2.318
217	9.4750	-3.6073	-5.3892	-4.8518	-4.3890	-4.0384	-4.8661	-4.6301	
218	9.8888	-6.9318	-6.5627	-6.8511	-6.7321	-6.8043	-6.9911	-6.7937	-7.052
219	9.5524	-5.9910	-5.6068	-5.0689	-5.6513	-5.2945	-6.2293	-6.0307	
220	8.9928	-5.0132	-4.4762	-4.7724	-4.6611	-4.7224	-4.8495	-4.6490	-5.103
221	8.6655	-2.7496	-3.4135	-2.8832	-3.2617	-2.8943	-3.7439	-3.5355	
222	8.1667	-2.6498	-2.2215	-2.5247	-2.4299	-2.4775	-2.5413	-2.3387	-2.799
223	7.6063	-0.2218	-0.2424	0.2790	-0.1243	0.2639	-0.5000	-0.2896	
224	7.3383	0.0170	0.4519	0.1417	0.2074	0.1782	0.1871	0.3908	-0.049
225	6.9611	2.7659	2.0525	2.5684	2.1384	2.5438	1.8462	2.0571	
226	6.4922	3.2653	3.7411	3.4238	3.4408	3.4378	3.5327	3.7363	3.304
227	6.1863	6.2083	5.3162	5.8256	5.3453	5.7783	5.1687	5.3791	
228	5.5598	7.7804	8.2691	7.9441	7.8728	7.9121	8.1199	8.3214	7.916
229	5.2072	11.3975	10.5197	11.0212	10.4337	10.9183	10.4395	10.6468	
230	4.8095	12.3762	12.8780	12.5469	12.3625	12.4521	12.7780	12.9762	12.610
231	4.2529	17.3611	17.3202	17.8139	17.0404	17.6068	17.2993	17.5004	
232	4.1213	17.6449	18.2164	17.8796	17.5355	17.6929	18.1596	18.3531	18.021

Table 2: Calculated root means square standard deviation (σ) using the different models.

Model	σ
GLM1	0.6199
GLM2	0.4527
MGLM1	0.4367
MGLM2	0.3245
CPPM	0.6255
CPPMT	0.5412

The root mean square standard deviation σ has also been calculated for the different models in order to compare the agreement between the experimental half-lives and theoretically calculated half-lives. The following formula has been used to compute the standard deviation (Akrawy et al., 2018):

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N \left[\left(\log_{10} T_{1/2,i}^{\text{Expt}} - \log_{10} T_{1/2,i}^{\text{Theory}} \right)^2 \right]} \quad (33)$$

where $T_{1/2,i}^{\text{Theory}}$ denote the half-lives obtained using the theoretical models and $T_{1/2,i}^{\text{Expt}}$ are the experimental half-lives.

The results of the standard deviation (σ) calculations using the different models are shown in Table 2 for the six models used in this study. The standard deviation values for the GLM1, GLM2, MGLM1, MGLM2, CPPM, and CPPMT, are 0.6199, 0.4527, 0.4367, 0.3245, 0.6255, and 0.5412, respectively. From these values, one observes that the modified Gamow-like model (MGLM2) gives the lowest standard deviation. This suggests that the modified Gamow-like model is the most suitable to describe the α -decay half-lives of the Thorium isotopes. Comparing the standard

deviations computed using the CPPM and CPPMT, it can be observed that the CPPMT gives better description of the α -decay half-lives than the CPPM. The use of the temperature-dependent proximity potential models has been shown, in recent times, to give better descriptions of the α -decay half-lives than the temperature-independent version. The calculated temperature (in MeV) in the CPPMT model is plotted against neutron number N in Figure 1. The highest temperature (about 0.92 MeV) is obtained for $N = 128$ (^{218}Th) which has the lowest half-life.

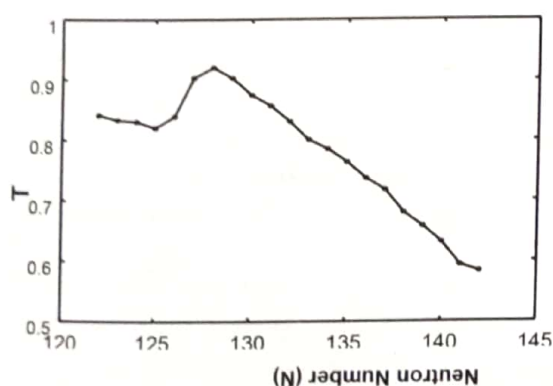


Figure 1: Plot of the calculated Temperature (in MeV) against Neutron number (N) for the Th isotopes.

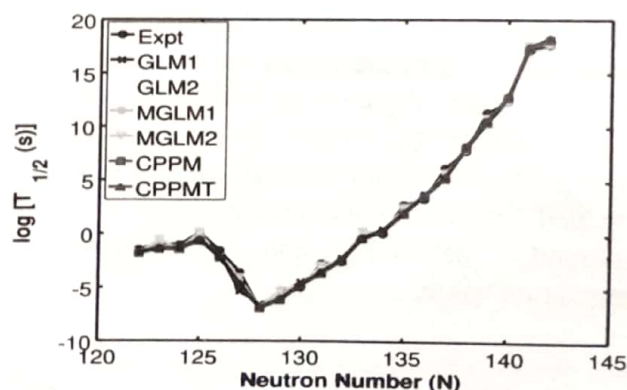


Figure 2: Comparison of the calculated α -decay half-lives of Th isotopes between the various models and experiment.

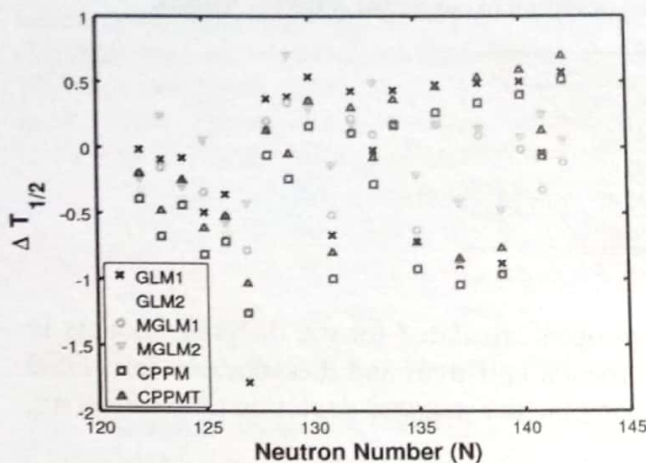


Figure 3: Plot of the ΔT against Neutron number (N) for the Th isotopes using the different models.

The calculated half-lives $\log[T_{1/2}(s)]$ using the six theoretical models have been plotted against the neutron number in Figure 2. The experimental values are shown in black circles. The minimum value of the half-lives is obtained at $N = 128$ which corresponds to ^{218}Th . This is due to the role of shell closure effects relative to the magicity of the neutron number. The difference between experimental and theoretical α -decay half-lives have also been calculated using the following equation (Akrawy et al., 2019):

$$\Delta T_{1/2} = \log_{10}[T_{1/2}^{\text{theor}}] - \log_{10}[T_{1/2}^{\text{expt}}] \quad (34)$$

This is plotted against neutron number in Figure 3 for all the models used in this study. It can be observed that, barring few exceptions, most of the points are near zero and within ± 0.5 . Figure 4 shows the plot of the experimental Q_α values against neutron number. The maximum value of the Q_α is at $N = 128$ (^{218}Th). This corresponds to the minimum half-life and maximum temperature values.

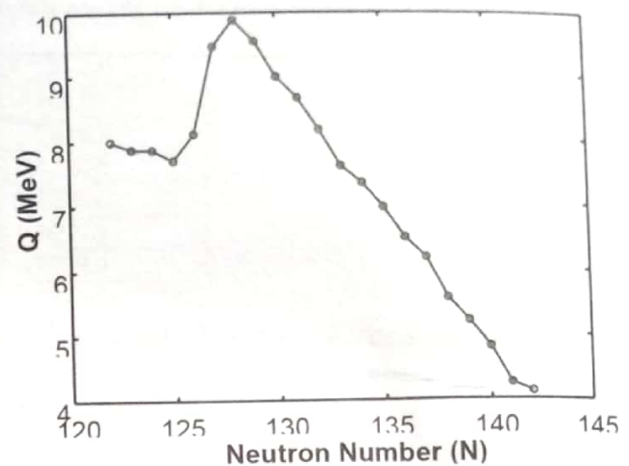


Figure 4: Plot of the Q_α against Neutron number (N) for the Th isotopes.

CONCLUSION

The theoretical study of the α -decay half-lives of $^{212-232}\text{Th}$ isotopes have been carried out using the Gamow-like model (GLM), modified Gamow-like model (MGLM), and Coulomb and proximity potential model (using the prox. 2010 potential). In the Coulomb and proximity potential model, both temperature-independent and temperature-dependent proximity potentials have been used. The use of temperature-dependent proximity potential is found to give better descriptions of the α -decay half-lives of the Thorium isotopes compared with the temperature-independent version. The computed standard deviation indicates that the modified Gamow-like model gives the least deviation. This suggests that the MGLM is the most suitable for describing the α -decay half-lives of the Thorium isotopes. In general, all the models give α -decay half-lives which are in good agreement with the available experimental data, with maximum standard deviation value less than 0.63.

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