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A control model of the operating head dynamics of Jebba hydropower system

Olalekan Ogunbiyi¹, Cornelius Thomas², Isaac O. A. Omeiza³, Jimoh Akanni⁴, and B. J. Olufeagba⁵. Electrical and Computer Engineering Department, Kwara State University, Malete, Nigeria¹ Electrical and Information Engineering Department, Achievers University, Owo, Nigeria² Electrical and Electronics Engineering Department, Faculty of Engineering, University of Ilorin, Nigeria^{3,4,5}

Abstract

Electricity availability in Nigeria has been very poor over the years, underscoring the need for a better approach for managing generating resources. This paper presents the development of a dynamical model of the operating head of Jebba hydroelectric power plants for system studies and control system design. The mathematical model of the plant was developed from flow continuity conditions, some model parameters were obtained from the source while others were estimated from observations and analysis of the measured data. The developed dynamical equation was validated by comparing the response produced with values obtained by measurement. Upon integrating the model equation in Microsoft EXCEL VBA® environment, a deviation of 2% from measured values was observed. Operators can therefore use the model as a decision support system, while control engineers can find the model directly applicable for optimal and robust control system design for the station.

Keywords: Control, Dynamical model, Hydropower, Inflow, Operating head.

Email: biyikan@gmail.com

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Introduction

Hydroelectric power plants generate electricity by converting kinetic and potential energy of water falling from a high potential, using different types of turbine to generate electricity (Kundur, 2010). Kinetic energy drives a shaft which is directly coupled to an alternator. It is one of the most efficient and cost-effective systems for producing electricity from renewable energy source (Okoro *et al.*, 2017). Taking together high reliability and available resources, hydropower stations are suitable for providing base load to national grid (Kundur, 2010; Gupta, 2011 and Gupta, 2013).

Jebba Hydroelectric Power Station (JHEPS), is located 103 km downstream of the Kainji Dam on the River Niger in Nigeria. The station is on 09°08'08"N, 04°47'16"E. It was commissioned on April 13, 1985, with a rated capacity of 578.4 MW and has six (6) fixed blade propeller type turbo-alternators each at 96.4 MW. The net operating head is about 27.6 m and its maximum flow per unit is around 380 m³/s. Each turbo-alternator is connected to a step-up transformer rated at 119 MVA maximum continuous rating but 103.50 MVA base load rating (Salami, 2007; Nwajinka *et al.*, 2013).

The Jebba Reservoir depends on discharge from the Kainji power station during the black flood but during the white flood some rivers downstream of Kainji flows into River Niger and hence into Jebba reservoir in addition to Kainji discharge. As a result, the Jebba reservoir is much smaller and must be operated in cascade with the Kainji reservoir. This arrangement imposes the

need for better water management if the units at Jebba are to operate efficiently all the year (Ale, 2014).

Problem associated with these subsections cannot be solved without proper mathematical model, resulting into differential or difference equation describing the dynamics (Kreyszig, 2011). This study tries to address the development of suitable dynamic model of operating head dynamics of JHEPS. It is intended to be used in this system study as well as optimal and robust control system design. Also considered is the development of numerical solutions for the resulting model equations, calibration of the models by determining parameters using measured data and hence study the system behaviour under changing conditions of inflow, load, head and unit availability.

Review of Related Work on JHEPS

A non-causal transfer function model of hydropower inflow using time series analysis was presented by Ale *et al.* (2011) and Nwobi-Okoye and Igboanugo (2012). The models were used to provide short term forecasts. It becomes even less reliable when the request for water by JHEPS is considered. The time series approach is a non-causal study which may veer away from reality if for example, the failures of turbo-alternators (TA) become very erratic.

Similarly, Abdulkadir *et al.* (2013) worked on modelling of hydropower reservoir variables for energy generation using the neural network approach. Reservoir variables and record of

energy generated over some years were used in training an artificial neural network.

A design concept of the complex hydro cascaded scheme was presented by Fustik and Kiteva (2006). It was proposed that a proper control architecture should involve measurement of real-time data using ICT.

The control model is not only used for system design, but it can also be the backbone of a decision support system. Such system was presented by Zhong *et al.* (2009) and previously designed ones are the Engineering Optimal Operation System of California, US central drainage area (CVP) and the water optimal operation system of the Tennessee Valley Organizations. They serve to assist the dispatcher of cascaded HEPS make an operational decision. The operators and management could therefore simulate operational decision, alternatives, and

assumptions and make an amendment to give a good precision.

Mathematical Modelling of the JHEPS

The formal design of control system requires suitable mathematical model for describing the process. Such model provides the simplest mathematical equation that can predict the response of the control system to an expected input (Kirk, 1998). Such models are a set of deterministic or stochastic, differential or difference equations. Hence the first step in the design of the optimal control system for JHEPS is the development of a set of differential equations that describe the system dynamics.

The basic information describing the structure of JHEPS can be seen in Table 1. The listed information are needed for the proper modelling of the dynamics of the system.

Table 1: Important Features of the KHEPS and JHEPS

Features	JHEPS
Location	- Latitude: 09°08'08"N, - Longitude: 04°47'16"E
Year of commissioning	1985: Units 1 to 6
Number of Units	6 Units
Installed Capacity	578.4 MW
Generator Rating	6 units of Kaplan turbine: 96.4 MW
Head (above sea level)	103 m (max) and 99 m (min)
Max. Net Head (m)	29.3 m
Rated Operating Head (m)	27.6 m
Min. Net Head (m)	24 m

Schematic Representation of JHEPS

Figure 1 Shows the schematic representation of the JHEPS. The variables affecting the reservoir

dynamics and power generation dynamics as a function of head are indicated.

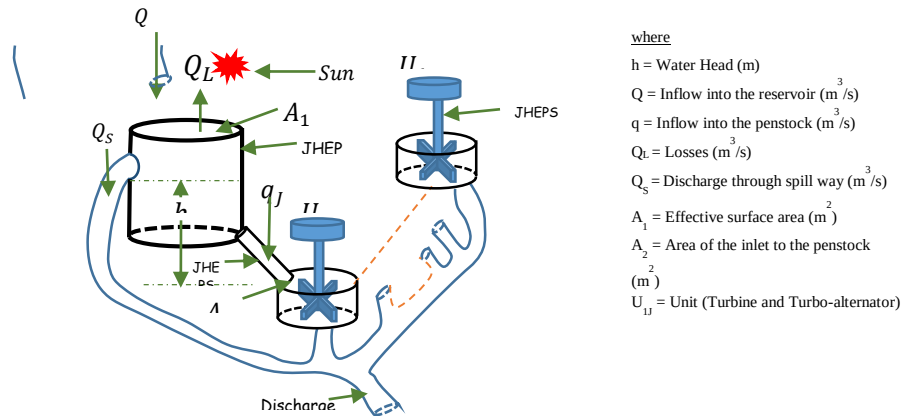


Figure 1: Schematic Representation of JHEPS

Electrical Power Output at JHEPS

Mechanical power (P_m) developed by a hydropower turbine of Figure 1 can be expressed as:

$$P_m = \rho g h q - \text{Losses in the penstock} \quad (1)$$

where ρ is the density of water in (kg/m^3), g is the acceleration due to gravity (m/s^2), h is the operating head of the reservoir (m); and q is the flow rate in (m^3/s).

Since the conversion efficiency (turbine efficiency and generator efficiency) η must of

necessity be less than 100%, the electrical power developed P_e can be expressed as:

$$P_e = \eta \rho g h q \quad (2)$$

and the flow rate q is related to the velocity by;

$$q = A_2 v \quad (3)$$

A_2 is the cross-sectional area of the penstock intake (m); and v is the velocity of water (m/s).

$$P_e = \eta \rho g h A_2 v \quad (4)$$

An expression for the velocity can be obtained from the Bernoulli's energy equation when applied to the input and output of the reservoir (Douglas *et al.*, 2005)

$$\frac{1}{2} \rho v_1^2 + \rho g h_1 + P_1 = \frac{1}{2} \rho v_2^2 + \rho g h_2 + P_2 \quad (5)$$

where P_1 and P_2 are atmospheric pressure values at the surface and the outlet, which in practice are approximately equal for Kaplan low head schemes and cancelled out in the equation (5). h_2 is the head at the outlet which equals zero.

Since the velocity of water at the head of the reservoir is much less than the velocity at the penstock outlet, v_2 is far greater than v_1 and equation (5) becomes:

$$\rho g h_1 = \frac{1}{2} \rho v_2^2 \quad (6)$$

$$v_2 = \sqrt{2 g h_1} \quad (7)$$

$$q = A_2 \sqrt{2 g h}, \text{ where } h \text{ is the same as } h_1 \text{ in equation (7)} \quad (8)$$

$$P_e = \sqrt{2} \eta \rho A_2 g^{3/2} h^{3/2} \quad (9)$$

$$P_e = \beta \eta h^{3/2} \quad (10)$$

$$\text{where } \beta = \sqrt{2} \rho A_2 g^{3/2}$$

JHEPS has 6 identical units of 96.4 MW fixed blade Kaplan turbo-alternators denoted by n_j , where n_j can take an integer value from 1 to 6. Hence, the electric power generated at JHEPS is represented by equation (11). The equation is based on the fact that efficiency of conversion is the same for the units.

$$P = \left(\sqrt{2} n_j \eta_j \rho A_2 g^{3/2} \right) h^{3/2} \quad (11)$$

$$P = \beta n_j \eta_j h^{3/2} \quad (12)$$

Equation (11) represents the power generated, indicating that it depends on the reservoir head and number of operating units. Consequently, it is clear that the dynamical model of hydropower station is determined by the dynamical consideration of both the alternator and reservoir operating head.

Dynamical Head Equations for the JHEPS

Consider the system represented in Figure 1 with Q as the inflow and q as the outflow;

$$A_1 \frac{dh}{dt} = (Q - Q_L - Q_s - q)t \quad (13)$$

$$A_1 \frac{dh}{dt} = Q - Q_L - Q_s - q = Q - Q_L - Q_s - A_2 v_2 \quad (14)$$

Equation (14) can be combined with equation (7) to give:

$$A_1 \frac{dh}{dt} = Q - Q_L - Q_s - \sqrt{2 g h} A_2 \quad (15)$$

$$\frac{dh}{dt} = -(\sqrt{2 g} A_1^{-1} A_2) h^{1/2} + A_1^{-1} (Q - Q_L - Q_s) \quad (16)$$

The head equation can be expressed as;

$$\frac{dh}{dt} = -\alpha h^{1/2} + \mu (Q - Q_L - Q_s) \quad (17)$$

where

$$\alpha = \sqrt{2 g} A_1^{-1} A_2$$

$$\mu = A_1^{-1}$$

The dynamical model for JHEPS with n_j number of units and $Q_n(t)$ (net inflow) can be expressed as:

$$\frac{d}{dt} h(t) = -n_j \alpha h^{1/2}(t) + \mu (Q(t) - Q_L(t) - Q_s(t)) \quad (18)$$

$$Q_n(t) = Q(t) - Q_L(t) - Q_s(t) \quad (19)$$

Equation (18) satisfies the standard representation of a model for a dynamical system represented by equation (20).

$$\frac{d}{dt} x(t) = f[x(t), u(t)] \quad (20)$$

where $x(t)$ represents the state of the system at time t , this corresponds to $h(t)$ in equation (18). $u(t)$ is the control input to the system at time t , corresponding to the net inflow $Q_n(t)$. f is a scalar-valued function of the state and control variable.

Following the development of equation (18), it is evident that the dynamical expression satisfies a nonlinear differential equation from which the closed form solution is not readily available, but it can be solved numerically. Different numerical techniques are discussed in detail in Hoffman (2001), Garreth (2015) and Kreyszig (2011). Both the Runge-Kutta numerical method and Adams-Moulton with Adams-Bashforth as predictor were experimented. The Adams-Moulton technique was chosen as the integrator in this work because it gives a result similar to that of Runge-Kutta technique and only one intermediate step need be computed instead of four steps in Runge-Kutta. The only disadvantage is that it is not self-starting; hence it must be started by another technique (Runge-Kutta in this case) or sufficient initial conditions are provided.

Model Calibration and Tuning Using Observations

While the model equations provide a framework for relating forces, dimensions and inputs to give a dynamical representation, the actual values that best relate the model output must be determined in a calibration / tuning process. The model format already takes care of the role of forces in the dynamics leaving the plant dimensions and inputs/ outputs.

As a result, the calibration process can be rationally prosecuted as two independent processes. It is noteworthy that physical dimensions do not affect the inputs/ outputs thus allowing for the separation of the calibration into two stages. These are the determination of the effective reservoir cross sectional area from observed and deduced inputs.

While certain parameters of the system are measurable, others such as the cross-sectional area of the reservoir and scroll casing are not measurable. Their values change with the head and opening of the scroll casing. To determine $\alpha = \sqrt{2g} A_1^{-1} A_2$ and $\mu = A_1^{-1}$, the two cross-sectional areas A_1 and A_2 were estimated from observation for tuning the model.

Estimation of Effective Surface Area (A_1) of the Reservoir

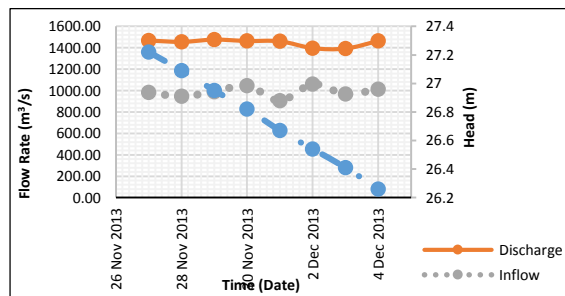
If observations of the inflow and head are studied such that a section of time where the behavior of the system is almost linear is selected, then;

$$\text{Volume} = A_1 * \Delta h = \int_{t_1}^{t_n} (Q_{(t)} - q_{(t)}) dt \quad (19)$$

$$A_1 = \frac{1}{\Delta h} \int_{t_1}^{t_n} (Q_{(t)} - q_{(t)}) dt \quad (20)$$

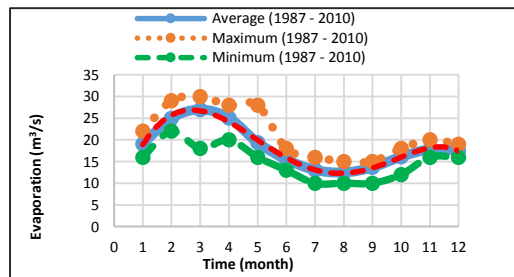
where $Q_{(t)}$ is the net inflow in (m^3/s), $q_{(t)}$ is the discharge in (m^3/s) and Δh is the change in head between time t_1 and t_n .

Observations from JHEPS were studied and a section where the reservoirs heads were almost linear for a given time were selected. The inflow, discharge and the head within the time frame were plotted in Figures 2. Equation (20) was



$$A_1 = 287,775,000 \text{ m}^2$$

Figure 2: Estimation of the Effective Surface Area



$$Q_{evp,j} = 0.0003t^6 - 0.0126t^5 + 0.0845t^4 + 1.8323t^3 - 25.699t^2 + 88.123t + 7.267 \quad (23)$$

Figure 4: Estimation of Evaporation Loss for JHEPS

solved to determine the effective surface area. It should be noted that the surface area varies with the head but the variation from the calculated effective value will be very minimal around the operating range of station. The effective surface area of JHEPS was estimated to be $A_1 = 287,775,000 \text{ m}^2$.

Estimation of Effective Area of the Scroll Casing

The effective area of the scroll casing can also be estimated from the observation. This is motivated by the fact that

$$q_i = A_{2i} n_i \sqrt{2gh_i}, \quad (21)$$

$$\text{for } i = 1, 2, 3, \dots, 365$$

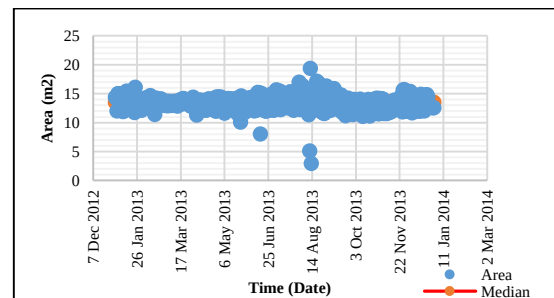
$$A_2 = \frac{1}{n_i \sqrt{2gh_i}} q_i, \quad (22)$$

where n_i represents the number of operating units on day i , q_i is the total station discharge on day i .

In estimating the effective area of the scroll casing (A_2) for JHEPS, the area was calculated per day using equation (22) from 1st of Jan. to 31st of Dec. 2013. The calculated values were plotted against time as shown in Figures 3. The median in each case was taken as and A_2 .

Estimation of Effective Evaporation Loss.

Evaporation loss used in this model was estimated from observations between 1985-2010. Monthly maximum, minimum and average evaporation loss were plotted in each case. The average value was mathematically modelled and presented in both Figure 4 and equations (23).



$$A_2 = 13.53289 \text{ m}^2$$

Figure 3: Estimation of Effective Area of the Scroll Casing

Results from Model Validation

The essence of computing a numerical solution for the resulting model and calibration of the parameters was to validate the model using measured data. This section presents the comparison between computed head from the model and the observed head from the data obtained. Other data used in the model include the inflows and the number of operating machines per day for the year 2013. Figure 5 presents the inflow into JHEPS in the year 2013 and Figure 6 presents the number of units operating per day. These two parameters, together with the evaporation losses were passed into the calibrated model to compute the

operating head of JHEPS. Comparisons of observed head with computed head for JHEPS in 2013 are presented in Figures 7a to 7d. The computed model is in two parts, one computed without evaporation losses while the other is computed with losses due to evaporation. The plot in Figure 7 represent the comparison between the model and measured date in the first quarter of the year. The plot for the model shows a good compliance with the measured data but a slight deviation can be observed for the model computed without evaporation. The evaporation loss is therefore significant in the performance of HEPS and the optimal use of the water should be put into consideration.

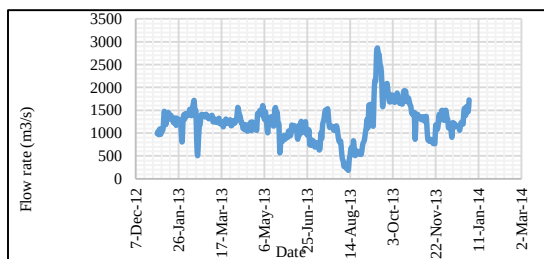


Figure 5: Inflows

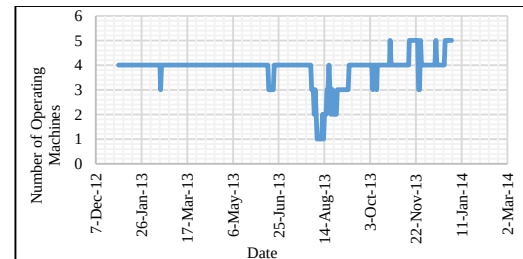
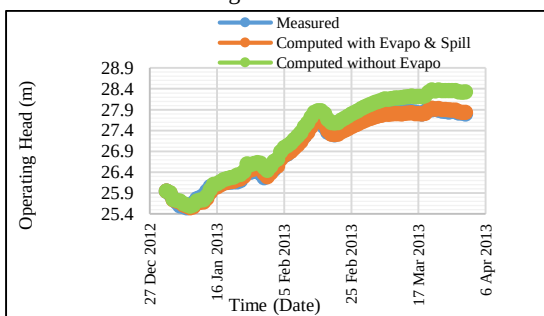
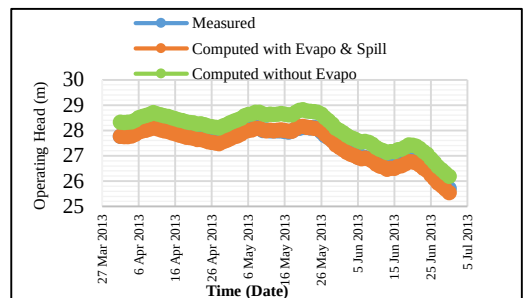


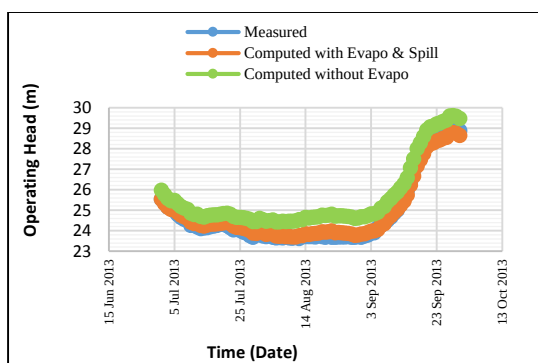
Figure 6: Machine State



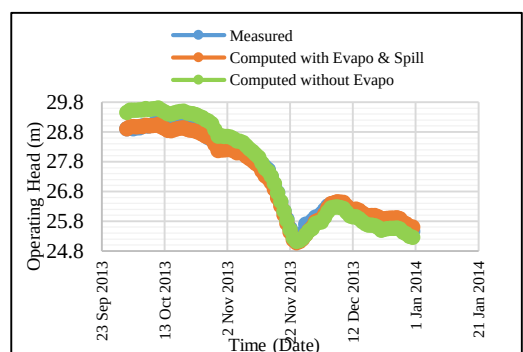
(a) Jan. - Mar.



(b) Apr. - Jun.



(c) Jul. - Sep.



(d) Oct. - Dec.

Figure 7: Comparison of Observed Head with Computed Head for JHEPS in 2013

The difference between the observed head and computed head was estimated in Figure 8 to be less than $\pm 1.5\%$. Numerous factors may be responsible for the deviation but a significant one is that the estimated evaporation losses may vary from the actual evaporation losses in 2013. Also,

the actual number of units operating per day may not reflect the value used in the model. A machine may not work for larger part of the day but in as much as it was operating as at the time the reading was taken, it will be assumed that it was operating for a whole day. These among others are

responsible for the slight deviation between the measured value and computed value.

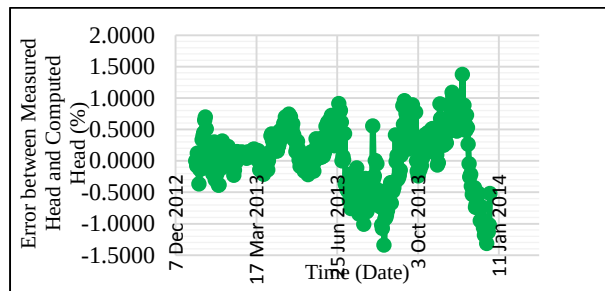


Figure.8: Error between Measured Head and Computed Head for JHEPS 2013

Conclusion

The limitation to the proper study of the operating head and design of appropriate control system for the JHEPS posed by the need for the proper control model of the system dynamics was solved in this research. The dynamical model was developed from flow continuity conditions while solution was obtained numerically. The model was developed so that when solved, they provide a basis for predicting reservoir conditions depending on the prevailing history of inflows, evaporation and power generation. Where the model parameters were not available, the estimation from observation becomes the option. This has been demonstrated in this work such that the effective reservoir surface area and effective cross-sectional area of the scroll casing were estimated from observations.

The work reported here addresses one of a plethora of problems that confront operators of cascaded hydropower stations.

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