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PAPER

Predictions of the alpha-decay half-lives of even–even superheavy nuclei using modified Gamow-like model

O K Azeez, W A Yahya  and A A Saeed

Department of Physics and Materials Science, Kwara State University, Nigeria

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5 April 2022**Abstract**

α -decay half-lives of superheavy nuclei have been studied using modified Gamow-like model and improved empirical formulas. The empirical formulas used in the study are Viola-Seaborg semi-empirical formula, universal decay law (UDL), Denisov-Khudenko formula (DK), Royer formula, Wang formula, AKRE formula, and new Ren B (NRB) formula. New values of the adjustable parameters in the modified Gamow-like model have been obtained using a least square fit scheme and input data from the NUBASE2020 database. We found that only two adjustable parameters are sufficient to describe the α -decay half-lives of heavy and superheavy nuclei. We also obtained new coefficients for the seven empirical formulas using similar procedure. The modified Gamow-like model (MGLM) and improved empirical formulas are then used to calculate the α -decay half-lives of measured superheavy nuclei with $Z = 100$ – 118 . The calculated root mean square errors indicate that the Royer, DK, and AKRE formulas are the best three empirical formulas for evaluating the α -decay half-lives of the superheavy nuclei. The five best performing empirical formulas, together with the MGLM, are then used to predict alpha-decay half-lives of unmeasured even–even superheavy nuclei with $Z = 120$ – 126 . The Q_α values used for the predictions have been taken from the tabulated Weizsäcker-Skyrme-4 + RBF (WS4 + RBF) Q_α values. The theoretical calculations suggest that $N = 184$ is a possible magic number, which is in agreement with previous theoretical calculations.

1. Introduction

α -decay is one of the most important types of radioactive decay, due of its ability to provide insight into the structure of a nucleus. It was discovered by Ernest Rutherford in 1899 [1], while investigating radioactivity in uranium nucleus. The α -decay process was first explained by Gamow [2], Condon and Gurney [3] as a quantum tunneling effect, which is regarded as the first attempt at the quantum description of nuclear phenomena. The Gamow theory explained that the α -decay was as a result of the quantum mechanical tunnelling of a charged α particle through the nuclear Coulomb barrier [4]. Since then, many theoretical models and empirical formulas have been employed in the investigation of α -decay half-lives of nuclei [4–19]. Some of the successful empirical formulas in the calculations of α -decay half-lives include Viola and Seaborg formula (VSS) [20], Brown formula [21], Royer formula [22, 23], Poenaru formula [24], Wang formula [25], Denisov-Khudenko (DK) formula [26], AKRE formula [27], Ren *et al* formulas [28], New Ren *et al* formulas [29], etc. The study of α -decay is important in the investigation of nuclei particularly heavy and superheavy nuclei (SHN) [30]. α -decay is a dominant decay mode for SHN, and it can provide nuclear structure information such as ground state properties and nuclear shell structure [31]. SHN are obtained from the fusion of lighter nuclei with the aid of particle accelerators. The SHN are located at the top right corner of the nuclear landscape. The study of the decay properties of SHN have been able to provide us with evidence that at the ambience of $Z = 114$, $N = 184$, enhanced stability can be achieved for SHN [32]. Although nuclei with $Z \leq 118$ have been obtained in the laboratory, nuclei with $Z > 118$ are yet to be experimentally synthesized [33]. As a result, there have been theoretical α -decay studies carried out on unmeasured SHN with $Z > 118$ to predict their half-lives [31, 33–36].

Geiger and Nuttall [37] introduced the first decay law to study α -decay half-lives. Later, Viola and Seaborg introduced semi-empirical relation for α -decay half-lives. In 1992, Brown proposed an empirical formula based on the experimental variation of logarithmic half-lives [21]. Royer in 2000 [22] formulated an empirical formula for α -decay half-lives using the liquid drop model which includes proximity effects. Also, Poenaru proposed the SemFIS formula for α -decay half-lives of SHN [24]. In 2015, Wang *et al* [25] obtained the α -decay half-lives by considering the ground state spin and parity of parent and daughter nuclei. There are phenomenological models, based on the Gamow-theory, introduced to study half-lives of α -decay. For example, Zdeb *et al* [4] presented a phenomenological model that is based on the Gamow theory to compute the α -decay half-lives of various isotopes (the model is termed Gamow-like model). In the Gamow-like model (GLM), the nuclear potential used is the square well potential, and the Coulomb potential is taken to be a uniformly charged sphere, while the centrifugal potential is ignored [4]. The model contains the nuclear radius constant as the adjustable parameter and was able to give good descriptions of the α -decay half-lives of 298 α -emitters. Moreover, the hindrance factor, which is due to the effect of an odd-proton/neutron, was also included. The hindrance factor, not present in the Gamow theory, is zero for even–even nuclei. The Gamow-like model gives better results of the α -decay half-lives due to the inclusion of the hindrance factor and the nuclear radius constant (as an adjustable parameter). The model, however, ignored the centrifugal potential.

The use of the Hulthen type of screened electrostatic potential in nuclear decay was first introduced in [38] to study proton radioactivity. Following its success, the Hulthen potential was employed in the modified Gamow-like model to study α -decay by Cheng *et al* [5] and later by [6]. The α -daughter nucleus potential in the modified Gamow-like model (MGLM) includes the Hulthen type of screened electrostatic Coulomb potential and the centrifugal potential. Unlike the Gamow-like model, the MGLM includes the effect of the centrifugal potential and electrostatic shielding. The MGLM contains, as adjustable parameters, the radius constant, a parameter which is related to the screened electrostatic barrier, and the hindrance factor. The model is, therefore, more effective for studying α -decay half-lives of nuclei. Recently, the Hulthen type of screened electrostatic potential was also used to study α -decay of heavy and SHN with $Z \geq 84$ [39]. The potential model has also been used to study cluster radioactivity [40].

The aim of this study is to calculate the α -decay half-lives of SHN using the modified Gamow-like model (MGLM) and improved empirical formulas. In this work, new values of the adjustable parameters in the MGLM are determined using a least square fit scheme and data from 516 α -emitters in the NUBASE2020 database [41–43]. Seven improved empirical formulas are also used in the study. They are the Viola-Seaborg Semi-empirical formula, universal decay law (UDL), Denisov-Khudenko formula (DK), Royer formula, Wang formula, AKRE formula, and new Ren B (NRB) formula. The results of the calculations are compared with available experimental data and other theoretical calculations. Moreover, in this study, α -decay half-lives of unmeasured even–even SHN with $Z = 120$ – 126 are also predicted using the MGLM and the top five performing empirical formulas. It should be noted that, although the daughter nuclei should be treated as deformed nuclei, the current study assumes the sphericity of the nuclei. The paper is organized as follows: In section 2, the theoretical models used in the study are presented. The results of the calculations are given in section 3, and finally the summary is given in section 4.

2. Theoretical framework

This section describes the theoretical models used in the study. We begin with the modified Gamow-like model, followed by the improved empirical formulas.

2.1. Modified gamow-like model (MGLM)

In the MGLM, the interaction between the daughter nucleus and α particle is given as [5, 6]:

$$V(r) = \begin{cases} -V_0, & 0 \leq r \leq R \\ V_H(r) + V_\ell(r), & r \geq R \end{cases}, \quad (1)$$

where the depth of the potential well $V_0 = 25A_1$, A_1 being the mass number of the α particle. The Hulthen type [44, 45] of screened electrostatic Coulomb potential ($V_H(r)$) is given by

$$V_H(r) = \frac{aZ_1Z_2e^2}{e^{ar} - 1}, \quad (2)$$

where Z_1 and Z_2 are, respectively, the charge numbers of the α and daughter nuclei, and a represents the screening parameter. The centrifugal potential (V_ℓ) is given as:

$$V_{\ell}(r) = \frac{\left(\ell + \frac{1}{2}\right)^2 \hbar^2}{2\mu r^2}, \quad (3)$$

where ℓ is the orbital angular momentum taken away by the α particle. The radius (R) is determined using:

$$R = r_0(A_1^{\frac{1}{3}} + A_2^{\frac{1}{3}}), \quad (4)$$

where r_0 is the radius constant.

The α -decay half-life can then be calculated via [6]:

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} 10^h, \quad (5)$$

where h is the hindrance factor. It is one of the adjustable parameters in the model. The value of h is zero for even–even nuclei. The values of the three adjustable parameters (a , r_0 , and h) in the model were determined in [5] to be

$$a = 7.8 \times 10^{-4}, r_0 = 1.14 \text{ fm}, h = 0.3455 \quad (6)$$

For odd–odd nuclei, they showed that $h_{np} = 2h$. The following formula is used to calculate the decay constant:

$$\lambda = \nu P_{\alpha} P, \quad (7)$$

where P_{α} is the α particle preformation probability at the nuclear surface. The best fitting results were obtained in [5] by setting $P_{\alpha} = 1$. In this study, the preformation factor P_{α} has been calculated via [46–48]:

$$P_{\alpha} = \frac{E_{f\alpha}}{E_t}, \quad (8)$$

where $E_{f\alpha}$ is the formation energy of the α particle, and E_t contains of $E_{f\alpha}$ and the interaction energy between the α particle and daughter nucleus. They are obtained via [46]:

$$\begin{aligned} E_{f\alpha} &= 3E_b(A, Z) + E_b(A - 4, Z - 2) - 2E_b(A - 1, Z - 1) - 2E_b(A - 1, Z) \\ E_t &= E_b(A, Z) - E_b(A - 4, Z - 2); \end{aligned} \quad (9)$$

$$\begin{aligned} E_{f\alpha} &= 3E_b(A - 1, Z) + E_b(A - 5, Z - 2) - 2E_b(A - 2, Z - 1) - 2E_b(A - 2, Z) \\ E_t &= E_b(A, Z) - E_b(A - 5, Z - 2); \end{aligned} \quad (10)$$

$$\begin{aligned} E_{f\alpha} &= 3E_b(A - 1, Z - 1) + E_b(A - 5, Z - 3) - 2E_b(A - 2, Z - 2) - 2E_b(A - 2, Z - 1) \\ E_t &= E_b(A, Z) - E_b(A - 5, Z - 3); \end{aligned} \quad (11)$$

$$\begin{aligned} E_{f\alpha} &= 3E_b(A - 2, Z - 1) + E_b(A - 6, Z - 3) - 2E_b(A - 3, Z - 2) - 2E_b(A - 3, Z - 1) \\ E_t &= E_b(A, Z) - E_b(A - 6, Z - 3), \end{aligned} \quad (12)$$

for even–even, even–odd, odd–even, and odd–odd nuclei, respectively, and $E_b(A, Z)$ denotes the binding energy of a nucleus with charge number Z and mass number A . The binding energies used here were taken from [42].

The penetration probability (P) is computed via:

$$P = \exp \left[-\frac{2}{\hbar} \int_R^b \sqrt{2\mu(V(r) - E_k)} dr \right], \quad (13)$$

where the reduced mass $\mu = \frac{A_1 A_2}{A_1 + A_2}$, and the kinetic energy of the emitted particle is obtained using

$E_k = \frac{Q_{\alpha}(A-4)}{A}$. The condition $V(b) = E_k$ is used to determine the classical turning point b . The assault frequency on the potential barrier and the radius of the parent nucleus are calculated using [5]:

$$\nu = \frac{\left(G + \frac{3}{2}\right) \hbar}{1.2\pi\mu R_0^2}, \quad (14)$$

and

$$R_0 = 1.28A^{\frac{1}{3}} - 0.76 + 0.8A^{-\frac{1}{3}}, \quad (15)$$

respectively. The global quantum number G is calculated using the Wildermuth quantum rule:

$$G = \begin{cases} 22 & N > 126 \\ 20 & 82 < N \leq 126, \\ 18 & N \leq 82 \end{cases} \quad (16)$$

where N is the neutron number for the parent nucleus.

2.2. Empirical Formulas

Here, we give brief descriptions of the seven empirical formulas used in the study. Many empirical formulas have been developed after the Geiger-Nuttall law. The new formulas include additional terms, such as nuclear isospin asymmetry and orbital angular momentum, to improve the prediction power. The seven empirical formulas used in this study are the Viola-Seaborg Semi-empirical formula, universal decay law (UDL), Denisov-Khudenko formula (DK), Royer formula, Wang formula, AKRE formula, and new Ren B (NRB) formula.

2.2.1. Viola-Seaborg Semi-empirical formula (VSS)

Viola and Seaborg [20] in 1966 proposed a formula based on Geiger-Nuttall law. This formula predicts logarithmic half-lives of α -decay by including the intercept parameter of linear dependence on the charge number of daughter nucleus:

$$\log_{10} T_{1/2}^{\text{VSS}}(s) = (aZ + b)Q_{\alpha}^{-1/2} + cZ + d + h_{\log}. \quad (17)$$

Here, Z is the charge number, Q_{α} is the α -decay energy of parent nucleus, while the coefficients a , b , c and $h_{\log}$ are the adjustable parameters. The values of the coefficients were determined to be $a = 1.66175$, $b = -8.5166$, $c = -0.20228$, $d = -33.9069$, and $h_{\log}$ is given by [49]:

$$h_{\log} = \begin{cases} 0 & \text{for even } Z \text{ and even } N \\ 0.772 & \text{for odd } Z \text{ and even } N \\ 1.066 & \text{for even } Z \text{ and odd } N \\ 1.114 & \text{for odd } Z \text{ and odd } N \end{cases}. \quad (18)$$

The above VSS formula is denoted as VSS1. In this study, we have used the modified VSS formula that depends on orbital angular momentum (ℓ) and isospin asymmetry ($I = (N - Z)/A$), given as

$$\log_{10} T_{1/2}^{\text{VSS}} = (aZ + b)/\sqrt{Q} + cZ + d + e(\ell(\ell + 1)) + fI, \quad (19)$$

This is denoted as VSS2.

2.2.2. universal decay law (UDL)

Qi et al [50] proposed a universal decay law to describe α decay and cluster decay modes starting from α -like R-matrix theory and the microscopic mechanism of the charged particle emission. This relates the half-lives of monopole radioactive decays with the Q_{α} values as well as its masses and charge. The formula is given as:

$$\log_{10} T_{1/2}^{\text{UDL}}(s) = a\chi + b\rho + c, \quad (20)$$

where

$$\chi = Z_1 Z_2 \sqrt{\frac{\mu}{Q_{\alpha}}}, \quad (21)$$

$$\rho = \sqrt{\mu Z_1 Z_2 (A_1^{-1/3} + A_2^{-1/3})}. \quad (22)$$

The parameters a , b , and c are adjustable parameters that are determined by fitting to experimental α and cluster decay half-lives. In this work, a modified form of the UDL that contains angular momentum term has been used. It is given by:

$$\log_{10} T_{1/2}^{\text{UDL}}(s) = a\chi + b\rho + c \frac{\sqrt{\ell(\ell + 1)}}{\rho} + d. \quad (23)$$

It has been adapted from [39, 51].

2.2.3. Denisov-Khudenko formula (DK)

The relationship between the half-life of alpha transition between the ground state of parent and daughter nuclei was proposed by Denisov and Khudenko [26]. The logarithm of the half-life was expressed as a function of the decay energy Q_{α} . The formula is given as:

$$\log_{10} T_{1/2}^{\text{DK}}(s) = a + b \frac{A^{1/6} \sqrt{Z}}{\mu_D} + \frac{cZ}{\sqrt{Q}} + \frac{d\sqrt{\ell(\ell + 1)}}{QA^{-1/6}} + e((-1)^{\ell} - 1), \quad (24)$$

where μ_D is given by:

$$\mu_D = \left[\frac{A}{(A - 4)} \right]^{1/6}, \quad (25)$$

The values of the adjustable parameters were given in [26].

2.2.4. Royer formula

Royer (in the year 2000) [52] proposed an analytical formula for α -decay half-lives which depends on charge number, mass number, and the decay energy Q_α . The formula is given as:

$$\log_{10} T_{1/2}^{\text{Royer}} = a + bA_6^{\frac{1}{6}}\sqrt{Z} + c\frac{Z}{\sqrt{Q}}, \quad (26)$$

where a, b, c are parameters obtained by fitting to experimental data [22]. Ten years later, Royer modified this α -decay empirical formula by adding angular momentum ℓ using the experimental data of 334 nuclei [23]. The modified Royer formula is given as:

$$\log_{10} T_{1/2}^{\text{Royer}} = a + bA_6^{\frac{1}{6}}\sqrt{Z} + c\frac{Z}{\sqrt{Q}} + \frac{dANZ}{Q}(\ell(\ell + 1))^{\frac{1}{4}} + eA[1 - (-1)^\ell], \quad (27)$$

The values of the adjustable parameters, obtained through least square fit scheme, are given in [23].

2.2.5. Wang formula

Wang *et al* [25] modified the Dong formula by adding the contribution of the angular momentum and a correction factor S which was set to 0.5. The Wang formula is given as:

$$\log_{10} T_{\frac{1}{2}}^{\text{Wang}} = aZQ^{-\frac{1}{2}} + bA^{-\frac{1}{6}}Z^{1/2} + c + \frac{d^{1-(-1)^\ell}\ell(\ell + 1)}{\sqrt{(A_2)(Z_2)A^{-2/3}}} + S, \quad (28)$$

where A_2 and Z_2 are the mass number and charge number of the daughter nucleus. The values of the parameters a, b, c , and d of the formula were already obtained in [25].

2.2.6. AKRE formula

Akrawy and Poenaru [27] postulated a new formula for calculating the α -decay half lives by adding the contribution of the isospin asymmetry term ($I = \frac{N-Z}{A}$) to the Royer formula [22] (in equation (26)). The AKRE formula is given as:

$$\log_{10} T_{1/2}^{\text{AKRE}} = a + bA_6^{\frac{1}{6}}\sqrt{Z} + \frac{cZ}{\sqrt{Q}} + dI + eI^2. \quad (29)$$

The parameters a, b, c, d , and e , obtained by fitting to experimental data, were given in [27].

2.2.7. New Ren B (NRB) formula

Ren *et al* [53] proposed an empirical formula for α and cluster decay of the form

$$\log_{10} T_{1/2}^{\text{RB}} = a\frac{\sqrt{\mu}Z_1Z_2}{\sqrt{Q}} + b\sqrt{\mu Z_1Z_2} + c, \quad (30)$$

where the reduced mass $\mu = A_1A_2/(A_1 + A_2)$. This formula is termed the Ren B formula. Akrawy *et al* [29] in 2019 modified the Ren B [53] formula by including the nuclear isospin asymmetry and angular momentum (ℓ) carried by the emitted α particle. The new formula is now termed as New Ren B (NRB) formula. NRB formula gave a better description of the α -decay half-lives than the Ren B formula, indicating the importance of angular momentum and nuclear isospin asymmetry in the α -decay formula. The New Ren B (NRB) is given by:

$$\log_{10} T_{\frac{1}{2}}^{\text{NRB}} = a\frac{\sqrt{\mu}Z_1Z_2}{\sqrt{Q}} + b\sqrt{\mu Z_1Z_2} + c + dI + eI^2 + f(\ell(\ell + 1)). \quad (31)$$

The values of a, b, c, d, e , and f for even–even, even–odd, odd–even and odd–odd are given in [29]. It should be noted that universal decay law (UDL) formula given in equation (20) is a generalization of the NRB formula [17].

3. Result and discussions

The goal of this study is to use improved empirical formulas and modified Gamow-like model to study the α -decay half-lives of SHN. We begin with the description of how improved parameters are obtained for the formulas. The experimental input data used in this study have all been extracted from the NUBASE2020 database [41–43]. The orbital angular momentum carried by the emitted α particle was calculated via the spin-parity selection rule given by [6, 29]:

Table 1. New Coefficients for Viola-Seaborg semi-empirical formula (VSS2).

Nuclei	Even-Even	Even-Odd	Odd-Even	Odd-Odd
<i>a</i>	1.510 89	1.569 71	1.503 14	1.622 51
<i>b</i>	5.828 17	1.765 20	9.782 36	−1.26664
<i>c</i>	−0.16771	−0.18751	−0.14834	−0.21629
<i>d</i>	−35.90051	−34.80403	−38.67487	−32.98868
<i>e</i>	0.065 91	0.037 19	0.030 92	0.008 59
<i>f</i>	−5.76059	−2.01126	−5.00000	1.484 12

Table 2. New Coefficients for universal decay law (UDL).

Nuclei	Even-Even	Even-Odd	Odd-Even	Odd-Odd
<i>a</i>	0.408 24	0.412 31	0.418 13	0.421 00
<i>b</i>	−0.41849	−0.40166	−0.42199	−0.39864
<i>c</i>	10.376 10	15.005 66	9.944 80	7.391 92
<i>d</i>	−21.93682	−23.39289	−22.68471	−24.28949

Table 3. New Coefficients for Denisov-Khudenko (DK) formula.

Nuclei	Even-Even	Even-Odd	Odd-Even	Odd-Odd
<i>a</i>	−25.82810	−26.64019	−26.62950	−28.25889
<i>b</i>	−1.13335	−1.08398	−1.10950	−1.04583
<i>c</i>	1.579 35	1.578 69	1.595 59	1.614 37
<i>d</i>	0.000 00	0.248 77	0.397 85	0.245 88
<i>e</i>	0.000 00	−0.45804	0.009 91	−0.02741

Table 4. New Coefficients for Royer formula.

Nuclei	Even-Even	Even-Odd	Odd-Even	Odd-Odd
<i>a</i>	−25.65929	−25.23931	−24.30631	−25.95298
<i>b</i>	−1.13643	−1.12001	−1.18607	−1.10881
<i>c</i>	1.578 91	1.560 67	1.578 67	1.584 83
<i>d</i>	0.000 00	7.0451×10^{-07}	9.7862×10^{-07}	9.7423×10^{-07}
<i>e</i>	0.000 00	0.001 81	6.3120×10^{-05}	$−9.6530 \times 10^{-05}$

$$\ell = \begin{cases} \Delta_J & \text{for } \pi_d = \pi_p \text{ and even } \Delta_J \\ \Delta_J + 1 & \text{for } \pi_d = \pi_p \text{ and odd } \Delta_J \\ \Delta_J & \text{for } \pi_d \neq \pi_p \text{ and odd } \Delta_J \\ \Delta_J + 1 & \text{for } \pi_d \neq \pi_p \text{ and even } \Delta_J \end{cases}. \quad (32)$$

Here, $\Delta_J = |J_p - J_d|$, J_d , π_d , J_p , π_p are the spin and parity values of the daughter and parent nuclei, respectively. In this study, new coefficients have been obtained for the empirical formulas by applying the least square fit scheme and using the data for 516 α emitters in the NUBASE2020 database, containing 180 even–even, 134 even–odd, 113 odd–even and 89 odd–odd nuclei. The logarithms of the α -decay half-lives were used as the output. The new values of the coefficients for VSS2, UDL, DK, Royer, Wang, AKRE, and NRB formulas are shown in tables 1–7, respectively. Moreover, the root mean square error (RMSE) calculated using the formula

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N [(\log_{10} T_{1/2,i}^{\text{theory}} - \log_{10} T_{1/2,i}^{\text{expt}})^2]}, \quad (33)$$

are 0.540 75, 0.547 55, 0.554 52, 0.559 75, 0.561 58, 0.589 16, 0.59251 for Royer, DK, NRB, VSS2, UDL, Wang, and AKRE formulas, respectively.

For the modified Gamow-like model, there are three adjustable parameters viz. the screening parameter (a), effective nuclear radius parameter (r_0), and hindrance factor (h). Initially, we obtained the new values of the parameters a and r_0 , while setting h to zero. The values obtained are shown in table 8. We then fixed the values of a and r_0 to determine the value of h , but we found that the value of h does not improve the results further. This

Table 5. New Coefficients for Wang formula.

Nuclei	Even-Even	Even-Odd	Odd-Even	Odd-Odd
<i>a</i>	−26.15930	−28.06349	−27.66732	−29.16054
<i>b</i>	−1.13643	−1.07560	−1.12399	−1.06569
<i>c</i>	1.578 91	1.604 48	1.623 92	1.640 75
<i>d</i>	0.000 00	1.024 94	0.839 34	0.633 39

Table 6. New Coefficients for AKRE formula.

Nuclei	Even-Even	Even-Odd	Odd-Even	Odd-Odd
<i>a</i>	−26.38382	−26.30827	−26.37629	−25.53477
<i>b</i>	−1.13162	−1.06644	−1.12706	−1.16056
<i>c</i>	1.581 96	1.573 94	1.636 19	1.595 01
<i>d</i>	9.099 33	−9.84377	−10.56125	1.421 34
<i>e</i>	−33.72191	48.682 63	28.381 62	20.052 39

Table 7. New Coefficients for New Ren B (NRB) formula.

Nuclei	Even-Even	Even-Odd	Odd-Even	Odd-Odd
<i>a</i>	0.409 31	0.409 76	0.420 30	0.413 47
<i>b</i>	−1.41905	−1.37380	−1.40927	−1.43801
<i>c</i>	−15.22275	−15.24427	−15.39427	−14.53357
<i>d</i>	11.109 51	−7.15632	−7.25867	1.704 85
<i>e</i>	−56.01198	19.001 65	7.135 75	1.272 81
<i>f</i>	0.000 00	0.035 42	0.029 83	0.008 55

Table 8. New parameters for the modified Gamow-like model (MGLM).

Nuclei	Even-Even	Even-Odd	Odd-Even	Odd-Odd
<i>a</i>	9.2838×10^{-4}	7.2790×10^{-4}	2.5044×10^{-5}	6.2647×10^{-4}
r_0 (fm)	1.189 28	1.177 08	1.200 79	1.172 70

means that the two parameters *a* and r_0 are sufficient in MGLM to describe α -decay half-lives of nuclei. This was also observed in [39]. The calculated root mean square error is 0.614 62.

To study α -decay of unmeasured SHN using MGLM as described above, the preformation factors are required. The preformation factor formula used here depends on binding energies. Since there are currently no experimental binding energy values available for SHN with $Z > 118$, we have decided to obtain an empirical formula for the calculation of the preformation factor instead of using the preformation factor described in equation (8). To achieve this, the half-lives of 516 nuclei were calculated using MGLM by setting $P_\alpha = 1$. The experimental preformation factors were then obtained using

$$P_\alpha = T_{1/2}^{\text{theory}} / T_{1/2}^{\text{expt}} \quad (34)$$

The logarithm of the preformation factors obtained were then fitted to the formula:

$$\log P_\alpha = c \sqrt{\frac{A_2 Z_2}{\sqrt{Q_\alpha}}} + d, \quad (35)$$

where Z_2 and A_2 are charge and mass numbers of the daughter nuclei, respectively. The values of the parameters obtained are $c = -0.02759 \pm 5.4473 \times 10^{-4}$ and $d = 1.48455 \pm 4.4878 \times 10^{-2}$.

Before the α -decay half-lives of unmeasured SHN are predicted, it is essential to study the performance of the theoretical models on measured SHN. We have decided to determine α -decay half-lives of measured SHN with $Z = 100 - 118$ using MGLM and the improved empirical formulas.

The calculated root mean square errors are shown in table 9. One can observe that the Royer formula gives the lowest RMSE (0.43055) followed by Denisov-Khudenko formula (0.44594), suggesting that they are the most suitable empirical formulas to determine α -decay half-lives of SHN.

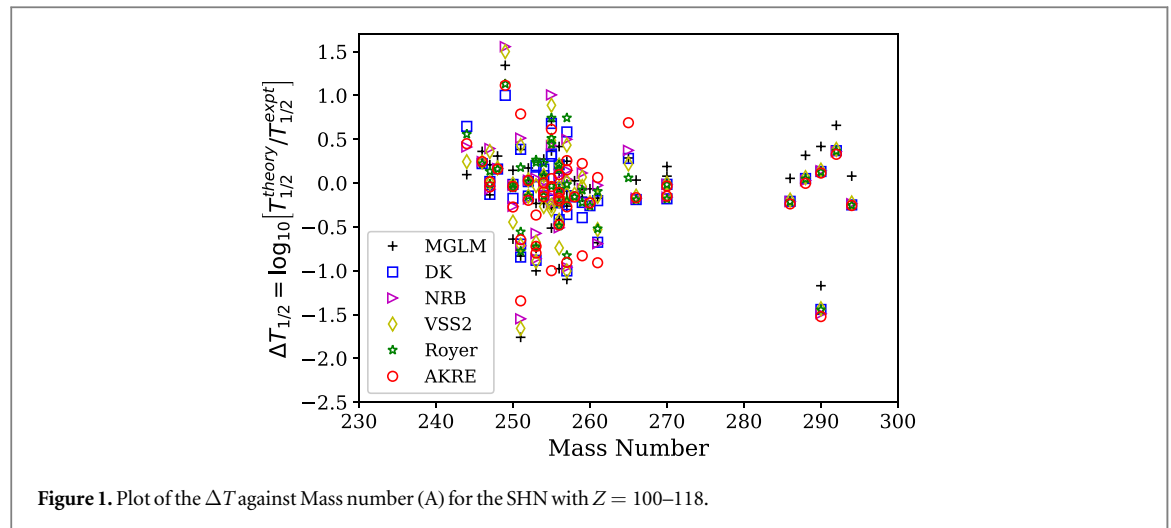


Table 9. Calculated root mean square errors (RMSE) for the modified Gamow-like model (MGLM) and empirical formulas for SHN with $100 \leq Z \leq 118$.

Model	RMSE	Model	RMSE
MGLM	0.552 17	Royer	0.430 55
DK	0.445 94	NRB	0.526 05
VSS2	0.530 53	UDL	0.535 67
Wang	0.545 82	AKRE	0.525 27

The values of the calculated α -decay half-lives for nuclei with $Z = 100 - 118$ using MGLM and the five best performing empirical formulas (viz. Royer, DK, AKRE, NRB, and VSS2) are shown in table 10. The first five columns of the table show, respectively, the mass number (A), charge number (Z), Q_α values, orbital angular momentum carried by the α particle (ℓ), and the experimental α -decay half-lives extracted from [43]. The sixth to eleventh columns display results of the theoretical calculations using MGLM, DK, NRB, VSS2, Royer, and AKRE formulas, respectively. The difference between the theoretical calculations and experimental data has been calculated using:

$$\Delta T_{1/2} = \log_{10}[T_{1/2}^{\text{theory}}/T_{1/2}^{\text{expt}}]. \quad (36)$$

This is plotted against mass number (A) in figure 1. One can observe from table 10 and figure 1 that the theoretical models give very good descriptions of the half-lives.

Next, we predict the α -decay half-lives of unmeasured SHN using MGLM and the best five empirical formulas viz. DK, Royer, NRB, VSS2, and AKRE formulas. Only even–even SHN with $Z = 120 - 126$ are considered here. An important input quantity required to predict the α -decay half-lives of the unmeasured SHN is the Q_α value. The Q_α values used were taken from [54], where the Weizsäcker-Skyrme-4+RBF (Q_α^{RBF}) formula was used. The results of the calculations are shown in table 11, and visualized in figure 2. In table 11, the first three columns are the mass number (A), charge number (Z), and Q_α^{RBF} . The fourth to ninth columns are the results obtained using MGLM, DK, NRB, VSS2, Royer, and AKRE formulas. When compared with previous theoretical calculations of Seyyedi [35] and Liu *et al* [31], our results are closer to the predictions of Liu *et al* than that of Seyyedi's predictions.

The plots in figure 2 show the half-lives of predicted SHN with $Z = 120, 122, 124$ and 126 . In figures 2(a)–(d), a drastic change in the values of the predicted half-lives can be observed for the nuclei at $N = 184$. This is as a result of the role of shell closure effects relative to the magicity of the neutron number of the parent nucleus. The magicity of the parent nucleus is indicated by a high half-life, while that of the daughter nucleus by a low half-life [7]. The theories therefore predict that $N = 184$ is a magic number, and this is consistent with the conclusion of [55–57].

Table 10. Calculated α -decay half-lives for nuclei with $Z = 100$ –118 using modified Gamow-like model and the five best performing empirical formulas.

A	Z	Q_α	ℓ	Expt.	MGLM	DK	NRB	VSS2	Royer	AKRE
246	100	8.3793	0	0.2181	0.5792	0.4397	0.4632	0.4597	0.4387	0.4624
247	100	8.2577	4	1.6852	1.8942	1.5562	2.0790	2.0452	1.6715	1.6407
248	100	7.9948	0	1.5378	1.8457	1.6975	1.7040	1.7102	1.6966	1.6984
249	100	7.7091	4	2.4638	3.8070	3.4648	4.0203	3.9638	3.5949	3.5786
250	100	7.5570	0	3.2695	3.4151	3.2538	3.2418	3.2577	3.2529	3.2311
251	100	7.4245	1	6.0253	4.2650	5.1806	4.4764	4.3669	5.2461	4.6808
252	100	7.1537	0	4.9610	4.9923	4.8128	4.7788	4.8081	4.8118	4.7640
253	100	7.1979	5	6.3345	6.1022	6.4061	6.3871	6.3053	6.5697	5.6120
254	100	7.3073	0	4.0671	4.3410	4.1511	4.0744	4.1526	4.1507	4.0705
255	100	7.2406	4	4.8588	5.5636	5.1909	5.8647	5.7464	5.3695	5.4710
256	100	7.0253	0	5.0658	5.4824	5.2744	5.1656	5.2707	5.2741	5.1627
257	100	6.8637	2	6.9396	6.6765	6.5812	6.9817	6.8116	6.7389	7.0905
244	101	8.9470	1	−0.4437	−0.3476	0.2028	−0.0326	−0.1993	0.1142	0.0055
247	101	8.7644	1	0.0755	−0.0590	0.0929	0.0055	0.0791	0.2090	0.0899
250	101	8.1554	2	2.8873	2.2475	2.7124	2.6218	2.4417	2.8323	2.6137
251	101	7.9634	1	3.4024	2.5692	2.7056	2.6856	2.6933	2.8452	2.7557
253	101	7.5731	1	5.0122	4.0104	4.1308	4.1478	4.1188	4.2849	4.2128
257	101	7.5571	1	5.1222	4.0224	4.1193	4.1605	4.1189	4.2950	4.2172
251	102	8.7517	0	−0.0160	0.3666	0.3689	0.4968	0.4120	0.1612	0.7723
252	102	8.5487	0	0.5622	0.7340	0.5784	0.5947	0.5981	0.5770	0.5908
253	102	8.4146	1	2.2337	1.4882	2.4328	1.6593	1.5586	2.4969	1.8687
254	102	8.2262	1	1.7550	1.8398	1.6095	1.6066	1.7550	1.6082	1.5989
255	102	8.4282	5	2.8476	2.3333	2.6534	2.6090	2.5421	2.8132	1.8475
256	102	8.5815	0	0.4663	0.5794	0.3962	0.3519	0.4204	0.3957	0.3568
257	102	8.4766	2	1.4597	1.3662	1.3165	1.6034	1.4778	1.4439	1.7159
259	102	7.8540	2	3.6665	3.5042	3.4453	3.7812	3.6272	3.5866	3.8893
254	103	8.8218	3	1.2237	0.9913	1.3823	1.1793	0.9558	1.4500	1.1211
255	103	8.5556	4	1.4941	1.8598	1.8000	1.9224	1.9788	1.9377	1.4540
256	103	8.8546	1	1.5162	0.5389	1.0993	1.0054	0.7760	1.0268	1.0378
255	104	9.0555	1	0.4896	0.2087	1.1696	0.3374	0.2349	1.2313	0.5403
256	104	8.9257	0	0.3282	0.2646	0.1021	0.1317	0.1234	0.1001	0.1260
257	104	9.0828	5	0.7481	0.9971	1.3312	1.2470	1.1808	1.4918	0.4761
258	104	9.1961	0	−0.5933	−0.5668	−0.7505	−0.7534	−0.7208	−0.7519	−0.7511
256	105	9.3361	2	0.3854	−0.0289	0.4468	0.2246	−0.0399	0.5411	0.2127
259	105	9.6188	5	−0.2924	−0.3676	−0.6872	−0.3560	−0.2256	−0.5146	−1.1228
260	106	9.9006	0	−1.7678	−1.8333	−2.0259	−1.9997	−1.9901	−2.0282	−1.9943
261	106	9.7137	2	−0.7292	−0.9018	−0.9300	−0.7551	−0.8815	−0.8239	−0.6662
261	107	10.5002	3	−1.8928	−2.5727	−2.5685	−2.5845	−2.4235	−2.4141	−2.8023
265	108	10.4703	0	−2.7077	−2.4217	−2.4266	−2.3346	−2.4897	−2.6483	−2.0191
266	108	10.3457	0	−2.4037	−2.3699	−2.5919	−2.5765	−2.5526	−2.5944	−2.5708
270	108	9.0699	0	0.9542	1.1432	0.9397	0.9398	0.9519	0.9370	0.9199
270	110	11.1170	0	−3.6882	−3.6095	−3.8679	−3.8494	−3.8212	−3.8708	−3.8378
286	114	10.3551	0	−0.6569	−0.6035	−0.8652	−0.8679	−0.8510	−0.8692	−0.8954
288	114	10.0765	0	−0.1851	0.1312	−0.1335	−0.1563	−0.1232	−0.1375	−0.1884
290	114	9.8562	0	1.9031	0.7315	0.4606	0.4136	0.4689	0.4568	0.3792
290	116	10.9968	0	−2.0458	−1.6282	−1.9139	−1.9049	−1.8980	−1.9185	−1.9320
292	116	10.7912	0	−1.7959	−1.1370	−1.4264	−1.4378	−1.4117	−1.4308	−1.4682
294	118	11.8673	0	−3.1549	−3.0756	−3.4040	−3.3903	−3.3816	−3.4090	−3.4116

4. Conclusion

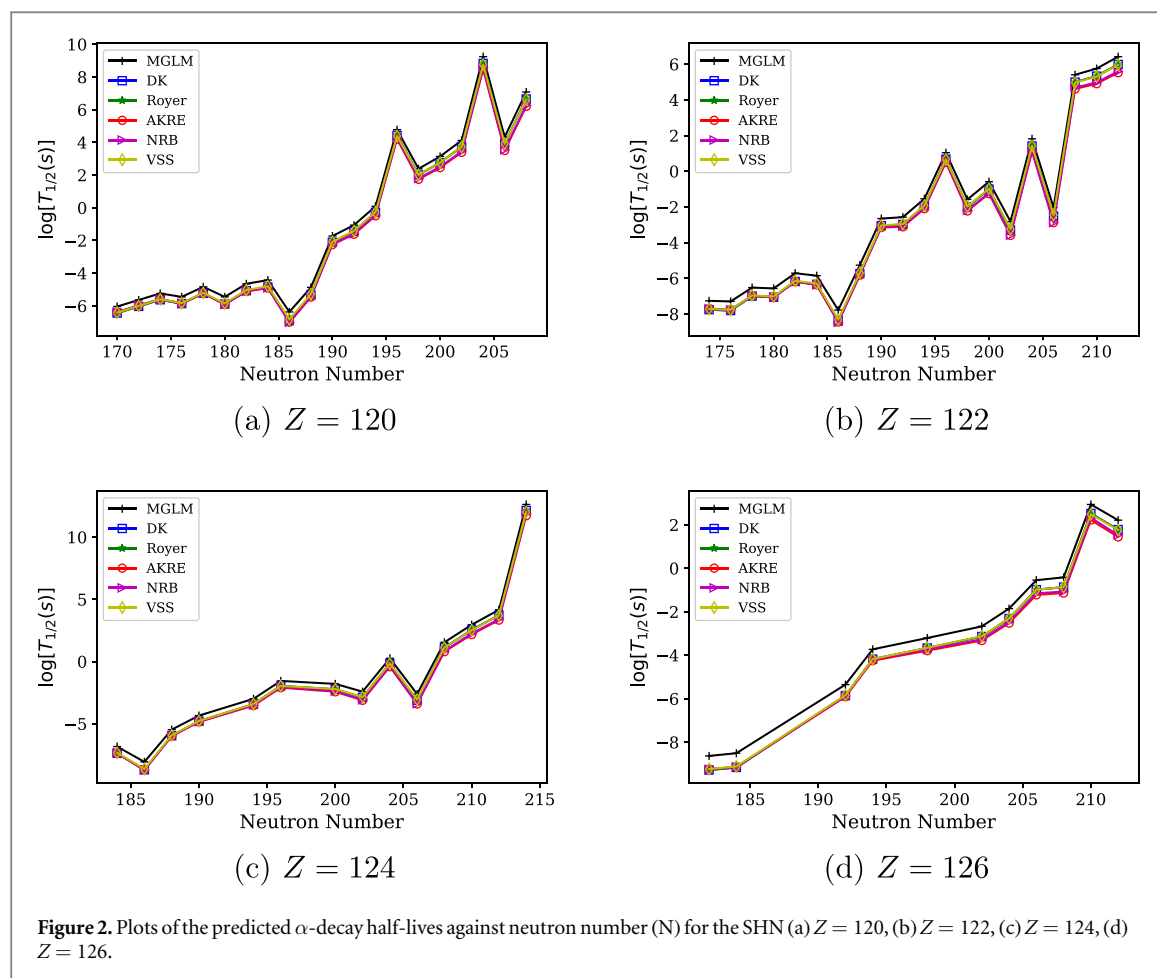
We have studied the α -decay half-lives of superheavy nuclei with the aid of modified Gamow-like model (MGLM) and improved empirical formulas. New parameter values for the MGLM and empirical formulas were determined using a least square fit scheme and input data from the NUBASE2020 database. We found that only two adjustable parameters (viz. the screening parameter (a) and effective nuclear radius parameter (r_0)) are sufficient to describe the α -decay half-lives of heavy and SHN. The MGLM and improved empirical formulas were then used to compute the α -decay half-lives of measured SHN with Z in the range 100–118. The results of the calculations showed that Royer, DK, and AKRE formulas were the best three empirical formulas to determine the α -decay half-lives of SHN. The MGLM was also found to give very good descriptions of the α -decay half-lives of the SHN. Following this, we predicted the half-lives from α -decay for unmeasured SHN with

Table 11. Prediction of α -decay half-lives of SHN with $Z = 120, 122, 124$, and 126 using MGLM, DK, NRB, VSS2, Royer, and AKRE formulas.

A	Z	Q^{RBF}	$\log [T_{1/2}(s)]$					
			MGLM	DK	NRB	VSS2	Royer	AKRE
290	120	13.6715	−6.0407	−6.4389	−6.3820	−6.4077	−6.4451	−6.3686
292	120	13.4370	−5.6379	−6.0306	−5.9791	−6.0006	−6.0366	−5.9745
294	120	13.2109	−5.2385	−5.6269	−5.5838	−5.5978	−5.6327	−5.5869
296	120	13.3124	−5.4510	−5.8627	−5.8389	−5.8263	−5.8681	−5.8417
298	120	12.9771	−4.8337	−5.2323	−5.2199	−5.1992	−5.2376	−5.2307
300	120	13.2896	−5.4516	−5.8907	−5.9085	−5.8446	−5.8955	−5.9123
302	120	12.8618	−4.6510	−5.0692	−5.1012	−5.0283	−5.0739	−5.1131
304	120	12.7358	−4.4216	−4.8442	−4.9004	−4.8010	−4.8487	−4.9130
306	120	13.7610	−6.3703	−6.8963	−7.0079	−6.8218	−6.9000	−6.9952
308	120	12.9405	−4.8694	−5.3369	−5.4603	−5.2764	−5.3407	−5.4613
310	120	11.4739	−1.7267	−2.1062	−2.2222	−2.0807	−2.1107	−2.2548
312	120	11.1919	−1.0628	−1.4408	−1.5839	−1.4176	−1.4451	−1.6190
314	120	10.7349	0.0880	−0.2823	−0.4483	−0.2675	−0.2867	−0.4906
316	120	9.1685	4.7781	4.4296	4.2848	4.3913	4.4241	4.1959
318	120	9.9081	2.3835	2.0137	1.7965	2.0124	2.0092	1.7417
320	120	9.6560	3.1379	2.7604	2.5098	2.7563	2.7559	2.4542
322	120	9.3459	4.1161	3.7299	3.4473	3.7204	3.7254	3.3889
324	120	7.9717	9.2419	8.8270	8.5648	8.7604	8.8214	8.4571
326	120	9.2687	4.3300	3.9199	3.5458	3.9222	3.9158	3.5034
328	120	8.4968	7.0781	6.6525	6.2652	6.6277	6.6479	6.2014
296	122	14.6658	−7.2647	−7.7580	−7.7144	−7.7186	−7.7643	−7.6966
298	122	14.6740	−7.2992	−7.8087	−7.7775	−7.7644	−7.8147	−7.7632
300	122	14.1928	−6.5223	−6.9996	−6.9725	−6.9613	−7.0056	−6.9703
302	122	14.2081	−6.5695	−7.0635	−7.0545	−7.0195	−7.0691	−7.0535
304	122	13.7102	−5.7170	−6.1797	−6.1792	−6.1423	−6.1852	−6.1891
306	122	13.7756	−5.8559	−6.3392	−6.3633	−6.2945	−6.3444	−6.3715
308	122	14.9138	−7.7834	−8.3952	−8.4711	−8.3182	−8.3996	−8.4549
310	122	13.4305	−5.2658	−5.7477	−5.8163	−5.6997	−5.7525	−5.8285
312	122	12.1370	−2.6542	−3.0522	−3.1151	−3.0338	−3.0575	−3.1555
314	122	12.0920	−2.5757	−2.9845	−3.0781	−2.9610	−2.9896	−3.1159
316	122	11.6344	−1.5453	−1.9403	−2.0537	−1.9243	−1.9453	−2.0994
318	122	10.6146	1.0384	0.6764	0.5620	0.6644	0.6709	0.4908
320	122	11.6326	−1.5837	−2.0052	−2.1925	−1.9756	−2.0097	−2.2272
322	122	11.2165	−0.5884	−1.0012	−1.2145	−0.9781	−1.0058	−1.2546

Table 11. (Continued.)

A	Z	Q^{RBF}	$\log[T_{1/2}(s)]$					
			MGLM	DK	NRB	VSS2	Royer	AKRE
324	122	12.1562	−2.8229	−3.3038	−3.5888	−3.2438	−3.3075	−3.5951
326	122	10.3077	1.8165	1.4132	1.1487	1.4184	1.4085	1.0945
328	122	11.7894	−2.0347	−2.5185	−2.8800	−2.4547	−2.5219	−2.8799
330	122	9.1601	5.4057	4.9942	4.6881	4.9669	4.9890	4.6082
332	122	9.0552	5.7521	5.3284	4.9799	5.3041	5.3233	4.9050
334	122	8.8636	6.4202	5.9834	5.5954	5.9581	5.9784	5.5223
308	124	14.6404	−6.8350	−7.3710	−7.3650	−7.3314	−7.3770	−7.3705
310	124	15.4081	−8.0659	−8.6981	−8.7304	−8.6352	−8.7035	−8.7213
312	124	13.8289	−5.4730	−5.9620	−5.9808	−5.9307	−5.9679	−6.0036
314	124	13.2190	−4.3441	−4.7962	−4.8246	−4.7750	−4.8021	−4.8602
318	124	12.5399	−2.9975	−3.4272	−3.4941	−3.4125	−3.4329	−3.5417
320	124	11.8853	−1.5574	−1.9595	−2.0381	−1.9584	−1.9654	−2.0999
324	124	11.9705	−1.7973	−2.2311	−2.3824	−2.2133	−2.2364	−2.4322
326	124	12.2366	−2.4205	−2.8842	−3.0816	−2.8508	−2.8891	−3.1183
328	124	11.1130	0.2597	−0.1563	−0.3552	−0.1529	−0.1617	−0.4187
330	124	12.3067	−2.6167	−3.1120	−3.3920	−3.0614	−3.1163	−3.4133
332	124	10.6213	1.5434	1.1205	0.8565	1.1208	1.1152	0.7912
334	124	10.1272	2.9574	2.5353	2.2474	2.5238	2.5299	2.1726
336	124	9.7353	4.1563	3.7283	3.4120	3.7081	3.7228	3.3308
338	124	7.5628	12.6256	12.1418	11.8961	12.0150	12.1342	11.7206
308	126	16.1380	−8.6316	−9.2802	−9.2501	−9.2395	−9.2871	−9.2359
310	126	16.0376	−8.5085	−9.1615	−9.1434	−9.1172	−9.1681	−9.1341
318	126	14.0252	−5.3507	−5.8583	−5.8743	−5.8380	−5.8648	−5.9096
320	126	13.1621	−3.7281	−4.1789	−4.1972	−4.1760	−4.1855	−4.2520
324	126	12.8867	−3.2073	−3.6656	−3.7340	−3.6572	−3.6718	−3.7909
328	126	12.6144	−2.6729	−3.1394	−3.2654	−3.1251	−3.1453	−3.3219
330	126	12.2349	−1.8559	−2.3113	−2.4609	−2.3017	−2.3172	−2.5231
332	126	11.6692	−0.5459	−0.9827	−1.1505	−0.9847	−0.9887	−1.2240
334	126	11.6089	−0.4187	−0.8654	−1.0701	−0.8619	−0.8712	−1.1393
336	126	10.3596	2.9361	2.5223	2.3254	2.4856	2.5158	2.2203
338	126	10.6009	2.2134	1.7811	1.5323	1.7622	1.7750	1.4432



$Z = 120$ – 126 using MGLM, Royer, DK, AKRE, NRB, and NRB formulas, with Q_α values taken from the tabulated Weizsäcker-Skyrme-4+RBF (WS4+RBF) Q_α values. The results of the calculations suggest that $N = 184$ is a possible magic number, which is in agreement with previous theoretical calculations.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

ORCID iDs

W A Yahya  <https://orcid.org/0000-0002-0105-1307>

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