

Modelling Of Accident Data In Nigeria: Poisson And Poisson-Gamma Regression

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Abstract: *Poisson and Poisson-gamma (Negative Binomial) regression models belong to the group of generalized linear models that are suitable to model count data. While the two regression types often give similar results, there can be differences in the effects of the covariates they estimated depending on whether the data is equi-dispersed, over-dispersed or under-dispersed. This work therefore demonstrates the behaviours of Poisson and Poisson-gamma regression models when the data is over-dispersed (variance greater than the mean). The two regression types were employed to model the number of people killed in road accidents as a function of the number of fatal, serious and minor accidents that occurred. Results from the two regression models indicated that number of people killed in road accidents is determined majorly by the number of fatal and serious cases of road accidents that occurred and not really by the number of minor accidents. The differences in the two regression models are described in the light of the over-dispersion in the data and the ability of the models to account for it. Road accident data of Kwara State, Nigeria for nine consecutive years from 2000 to 2008, primarily collected by the Nigeria Police Force (NPF), were employed in this study.*

Keywords: *Poisson and Poisson-gamma (Negative Binomial) regressions, Over-dispersion, Under-dispersed, Equi-dispersion, Generalized Linear Model, Accident data.*

I. INTRODUCTION

A count refers to the number of specified events of interest that occur in a given interval of time. By this

definition, count data consist of only non-negative integers. The specified event can include any behavior of interest, and counts are utilized frequently in econometrics and medicine, among others. Common examples of count data include number of crimes in a location, number of child births, patients hospitalized (Logan, Radcliffe, & Smith-Whitley, 2002), frequency of adolescent alcohol use (Audrain-McGovern, Rodriguez, Tercyak, Neuner & Moss, 2006), and number of childhood injuries (Morrongiello, Ondejko, & Littlejohn, 2004; Schwebel, Brezaussek & Ramey, 2004). Distributions of such data violate fundamental assumptions of many commonly used multivariate statistical techniques (such as ordinary least squares regression), leading to results that do not accurately reflect the observed data (Hammer & Landau, 1981). Fairly recently, statistical techniques that overcome these problems have been developed (Lambert, 1992; Hall, 2000). Although common, analysis of count outcomes presents unique challenges (Atkins & Gallop, 2007). When target behaviors are relatively rare, the resulting distributions are highly skewed. Such distributions violate fundamental assumptions of Ordinary Least Square regression, most notably normality of residuals. As a consequence; resulting sample statistics differ from true population parameters (Hammer & Landau, 1981), and the skew can lead to inaccurate standard errors and an increase in Type I or Type II error rates (Gardner, Mulvey, & Shaw, 1995).

Models for count data have been prominent in many branches of the recent applied literature, for example, in health, economics, management and industrial organization. The foundational building block in this modeling framework is the Poisson regression model.

However, because of its implicit restriction on the distribution of observed counts that the variance of the random

variable is constrained to equal the mean; researchers generally employ more general specifications such as the Negative Binomial (NB) model which is the standard choice for a basic count data model (Cameron and Trivedi, 1986).

The goal of the present work is therefore to illustrate the use of these techniques by offering a practical demonstration using prospective data from road accident. Some statistical tests of the two functional forms of regression models were used.

The study is organized as follows: section two will detail the theoretical background of Poisson and Poisson-Gamma regression. The application of the models is discussed in section three. Section four and section five contain some discussions and conclusion respectively.

II. THEORETICAL BACKGROUND OF POISSON AND POISSON GAMMA REGRESSION

This section comprises of theoretical background of the models. The central focus of the investigation was the choice of better model in accident data.

A. THE POISSON REGRESSION

Then the observed count, Y_i , is assumed to follow a Poisson distribution

$$P(Y_i = y_i) = \frac{e^{-\lambda_i} \lambda_i^{y_i}}{y_i!},$$

$$y_i = 0, 1, 2, \dots \text{ and } i = 1, 2, \dots, n$$

a. ASSUMPTIONS OF POISSON MODEL

- ✓ Assume $y_i \sim P(\lambda_i)$; response variable Y_i has a Poisson distribution; the expected count of Y_i , $E(Y_i) = \lambda_i$.
- ✓ The conditional variance equals conditional mean, that is,
 $E(Y_i) = \text{Var}(Y_i) = \lambda_i$.
- ✓ $\ln \lambda_i = \beta_0 + \beta_1 x_{1i} + \dots + \beta_{pi}$, for p -explanatory variables.
- ✓ $(y_i/x_i), i = 1, \dots, n$ are independently distributed.

b. POISSON MODEL'S PARAMETERS ESTIMATION

In the Poisson framework, the density function;

$$f(y_i) = \frac{\lambda_i^{y_i} e^{-\lambda_i}}{y_i!}, \quad \text{for } y_i = 0, 1, \dots \text{ and } \lambda_i > 0$$

The log-likelihood function is given as

$$\ell(\lambda_i, \omega; y_i) = \sum_{i=1}^n \frac{[y_i \lambda_i - b(\lambda_i) + c(y_i, \omega)]}{a_i(\omega)}, \text{ that is, as}$$

exponential family, which is expressed in terms of mean, λ_i and dispersion parameter ω . Since the Poisson model is a member of the exponential family of distribution, where

$$\lambda_i = \exp(X_i' \beta)$$

$$\ln[f(y_i; \lambda_i)] = \ln \left(\frac{e^{-\lambda_i} \lambda_i^{y_i}}{y_i!} \right) = [-\lambda_i + y_i \ln(\lambda_i) - \ln(y_i!)]$$

$$= \left[-e^{X_i' \beta} + y_i X_i' \beta - \ln(y_i!) \right] \quad \dots 2.1$$

The systematic component is the usual linear predictors, that is

$$\ln \lambda_i = \beta_0 + \beta_1 x_{1i} + \dots + \beta_{pi} = X_i' \beta$$

The usual link for Poisson regression is the canonical link for the log-linear model, which is logarithm. The model is said to have log link function, that is, the function that links the mean to the predictor. Negative value is not appropriate for Poisson mean, so $\lambda_i > 0$. Thus the Poisson regression model is ultimately $y_i \sim \text{Poisson}(e^{X_i' \beta})$, where

$$X' = [1 \quad x_1 \quad x_2 \quad \dots \quad x_p],$$

$$\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{bmatrix}$$

The likelihood function is given as;

$$L(\lambda_i; y_i) = \prod_{i=1}^n \frac{\lambda_i^{y_i} e^{-\lambda_i}}{y_i!}$$

The log-likelihood is

$$\ell(\lambda_i; y_i) = \sum_{i=1}^n [-\lambda_i + y_i \ln(\lambda_i) - \ln(y_i!)]$$

$$= \sum_{i=1}^n [-e^{X_i' \beta} + y_i \ln(e^{X_i' \beta}) - \ln(y_i!)]$$

$$= \sum_{i=1}^n [-e^{X_i' \beta} + y_i X_i' \beta - \ln(y_i!)]$$

$$\frac{\partial \ell(\lambda_i; y_i)}{\partial \beta} =$$

$$\sum_{i=1}^n [-e^{X_i' \beta} + y_i] X_i = 0 \quad \dots 2.2$$

$$\frac{\partial^2 \ell(\lambda_i; y_i)}{\partial \beta \partial \beta'} =$$

$$\sum_{i=1}^n [-e^{X_i' \beta}] X_i X_i' = 0 \quad \dots 2.3$$

The equations 2.2 and 2.3 are non linear in the parameter vector β , thus, an iterative procedure is necessary to obtain the maximum likelihood estimator for the vector of parameters, β .

In this regard, the l^{th} iterative estimate for β is obtained as;

$$\beta^l = \beta^{l-1} - H^{-1} S_{\beta^{l-1}}$$

where H is the Hessian matrix obtained from;

$$H = \frac{\partial^2 \ell(\lambda_i; y_i)}{\partial \beta \partial \beta'} \quad \text{and}$$

$$S_\beta = \frac{\partial \ell(\lambda_i; y_i)}{\partial \beta} \quad \text{as implemented by Jolayemi}$$

(1990) using Newton-Raphson iterative method.

Maximum likelihood estimates of β are generally not available in closed form, but can be obtained via an algorithm known as iteratively weighted least squares (IWLS). Statistical Analysis Software (SAS) can also be used to carry out Newton-Raphson method for solving the equations.

c. ADVANTAGES OF THE POISSON MODEL

When equi-dispersion condition is maintained, Poisson regression model is more appropriate for count data. Another advantage is that the constancy of sums is maintained (the sum of the predicted values equals the sum of the input values).

d. CHALLENGE OF THE POISSON MODEL

Poisson regression may underestimate the standard error and over-state the significance of the regression parameters when the data is over-dispersed, consequently giving misleading inference about the regression parameters.

B. THE NEGATIVE-BINOMIAL MODEL

The density of Poisson Gamma distribution, which is reparameterisation of Poisson and Gamman distributions is defined as:

$$P(Y = y) = \frac{\Gamma(y+k)}{\Gamma(k)\Gamma(y+1)} \left(\frac{k}{k+\lambda}\right)^k \left(\frac{\lambda}{k+\lambda}\right)^y, \quad y = 0, 1, 2, \dots$$

with expected value $E(Y_i) = \lambda_i$ and variance $V(Y_i) = \lambda_i + \lambda_i^2/k$, where, k is the *inverse dispersion parameter* (McCullagh and Nelder, 1989) and $\omega = 1/k$ is the *dispersion multiplier*. Unless k is large, the variance increases more rapidly with the mean than the variance of a Poisson variable. Making the expected value of Y_i a random variable incorporates additional variation among observed counts for observations that share the same values of the explanatory variables and consequently have the same linear predictor, $\ln(Y_i)$.

The model is $\lambda_i = \exp(X' \beta + \varepsilon_i)$
 $= \exp(X' \beta) \cdot \exp(\varepsilon_i) \quad \text{for } i = 1, 2, \dots, p,$
 where $X' = [1 \ x_1 \ x_2 \ \dots \ x_p]$,

$$\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{bmatrix}$$

and expected value of ε_i is 0. The $\exp(\varepsilon_i)$ is assumed to follow the gamma distribution with a mean equal to 1 and a variance equal to $\omega = 1/k$ where ω is a parameter that is greater than zero (Lord, 2006; Cameron & Trivedi, 1998).

a. POISSON-GAMMA MODEL'S PARAMETERS ESTIMATION

$Y_i \sim NB(\lambda_i, \omega)$, where ω is dispersion parameter ($\frac{1}{k} = \omega$) and k is inverse dispersion parameter.

$f(y_i) = \binom{y_i+k-1}{k-1} P^k (1-P)^{y_i}$, which is parameterized as the mixture of Poisson and Gamma distribution as;

$$f(y_i) = \binom{y_i+k-1}{k-1} \left(\frac{k}{k+\lambda_i}\right)^k \left(\frac{\lambda_i}{k+\lambda_i}\right)^{y_i},$$

where $P = \left(\frac{k}{k+\lambda_i}\right)$ and $E(Y_i) = \frac{k(1-P)}{P}$

Substitute for P ,

$$E(Y_i) = \lambda_i$$

$$\begin{aligned} \text{while } \text{Var}(Y_i) &= \frac{k(1-P)}{P^2} \\ &= \frac{k \left(\frac{\lambda_i}{k+\lambda_i}\right)}{\left(\frac{k}{k+\lambda_i}\right)^2} \\ &= \frac{k \lambda_i (k+\lambda_i)^2}{k^2 (k+\lambda_i)} \\ &= \frac{\lambda_i (k+\lambda_i)}{k} \\ &= \lambda_i + \omega \lambda_i^2 \end{aligned}$$

On the account of over-dispersion k is restricted to be positive such that $(Y_i) > \lambda_i$.

$$\begin{aligned} \therefore f(y_i) &= \frac{(y_i+k-1)(y_i+k-2) \dots (k)(k-1)!}{(k-1)! y_i!} \left(\frac{k}{k+\lambda_i}\right)^k \left(\frac{\lambda_i}{k+\lambda_i}\right)^{y_i} \\ &= \frac{(y_i+k-1)(y_i+k-2) \dots (k)(k-1)!}{(k-1)! y_i!} \left(\frac{k}{k+\lambda_i}\right)^k \left(\frac{\lambda_i}{k+\lambda_i}\right)^{y_i} \\ &= \frac{(y_i+k-1)(y_i+k-2) \dots (k)}{y_i!} \left(\frac{k}{k+\lambda_i}\right)^k \left(\frac{\lambda_i}{k+\lambda_i}\right)^{y_i} \end{aligned}$$

Recall that $\ln(\lambda_i) = X_i' \beta$

The likelihood function is given as

$$L(\lambda_i, k; y_i) = \prod_{i=1}^n \frac{(y_i+k-1)(y_i+k-2) \dots (k)}{y_i!} \left(\frac{k}{k+\lambda_i}\right)^k \left(\frac{\lambda_i}{k+\lambda_i}\right)^{y_i}$$

The log-likelihood equation is

$$\begin{aligned} \ell(\lambda_i, k; y_i) &= \sum_{i=1}^n \left\{ \sum_{j=0}^{y_i-1} \ln(k+j) - \ln(y_i!) + k \ln(k) + y_i \ln(\lambda_i) - (k+y_i) \ln(k+\lambda_i) \right\} \\ \frac{\partial \ell}{\partial k} &= \sum_{i=1}^n \left\{ \sum_{j=0}^{y_i-1} (k+j)^{-1} + k \cdot k^{-1} + \ln(k) - \ln(k+\lambda_i) - (k+y_i)(k+\lambda_i)^{-1} \right\} = 0 \\ &= \sum_{i=1}^n \left\{ \sum_{j=0}^{y_i-1} (k+j)^{-1} + 1 + \ln(k) - \ln(k+\lambda_i) - (k+y_i)(k+\lambda_i)^{-1} \right\} = 0 \end{aligned}$$

$$\frac{\partial^2 \ell}{\partial k^2} = \sum_{i=1}^n \left\{ \left[-\sum_{j=0}^{y_i-1} (k+j)^{-2} \right] + k^{-1} - (\lambda_i + k)^{-1} - (\lambda_i + k)^{-1} + (k + y_i)(\lambda_i + k)^{-2} \right\} = 0$$

$$= \sum_{i=1}^n \left\{ \left[-\sum_{j=0}^{y_i-1} (k+j)^{-2} \right] + k^{-1} - 2(\lambda_i + k)^{-1} + (k + y_i)(\lambda_i + k)^{-2} \right\} = 0$$

$$\frac{\partial^2 \ell}{\partial k \partial \beta} = \sum_{i=1}^n \left\{ x_i \left(\frac{(k + y_i)}{(\lambda_i + k)^2} \right) - (\lambda_i + k)^{-1} \right\} = 0$$

$$\frac{\partial \ell}{\partial \beta} = \sum_{i=1}^n \left\{ x_i \left(\frac{y_i}{\lambda_i} - \frac{(k + y_i)}{(\lambda_i + k)} \right) \right\} = 0$$

$$\frac{\partial^2 \ell}{\partial \beta \partial \beta'} = \sum_{i=1}^n \left\{ x_i x_i' \left(\frac{y_i}{\lambda_i^2} - \frac{(k + y_i)}{(\lambda_i + k)^2} \right) \right\} = 0$$

The Newton-Raphson Algorithm steps are as follow;

✓ Set $k=(0,1)$, iterate to obtain estimate for β ;

$$\beta^l = \beta^{l-1} - H_{\beta}^{-1} S_{\beta^{l-1}}$$

✓ Set $\beta=(1,0,0)$ and iterate to obtain estimate of k ;

$$k^l = k^{l-1} - H_k^{-1} S_{k^{l-1}}$$

✓ Use result from steps 1 and 2 as starting values to obtain estimate of k and β ;

$$\begin{bmatrix} \beta \\ k \end{bmatrix}^l = \begin{bmatrix} \beta \\ k \end{bmatrix}^{l-1} - H_{\beta}^{-1} S_{\beta^{l-1}}$$

where H is the Hessian matrix obtained from;

$$H_{\beta} = \frac{\partial^2 \ell}{\partial \beta \partial \beta'}, \quad \text{and} \quad S_{\beta} = \frac{\partial \ell}{\partial \beta}$$

$$H_k = \frac{\partial^2 \ell}{\partial k^2}, \quad S_k = \frac{\partial \ell}{\partial k}$$

$$H_{\beta k} = \begin{bmatrix} H_{\beta} & \frac{\partial^2 \ell}{\partial k \partial \beta} \\ \frac{\partial^2 \ell}{\partial k \partial \beta} & H_k \end{bmatrix}, \quad S_{\beta k} = \begin{bmatrix} S_{\beta} \\ S_k \end{bmatrix}$$

Similar to the case of Logistic regression and Poisson regression, the *maximum likelihood estimators* (MLEs) for $(\beta_0, \beta_1 \dots \text{etc.})$ are obtained by finding the values that maximizes log-likelihood. In general, there are no closed-form solutions, so the Maximum Likelihood estimates are obtained by using iterative algorithms such as *Newton-Raphson* (NR), *Iteratively reweighted least squares* (IRWLS), etc.

b. ADVANTAGES OF THE POISSON GAMMA MODEL

The main advantage of the Poisson-Gamma model over the Poisson is that it, most often than none, performs more effectively and appropriately when over-dispersion exists. Mathematically, it separates out the assumptions of the mean (Poisson) from that of the variance; that is dispersion (Gamma). This is very useful for modeling over-dispersed data.

c. REMARK

Poisson-Gamma is not necessary when equi-dispersion is maintained in count data. However, wherever count data are in focus, over-dispersion must be tested and when not present Poisson should be used.

C. INFERENCES

The usual tools from the basic statistical inference are valid in Generalized Linear Model.

a. OVERALL GOODNESS-OF-FIT STATISTICS OF THE MODEL

✓ Pearson chi-square statistic,

$$X^2 = \sum_{i=1}^n \left(\frac{(y_i - \lambda_i)^2}{v(y_i)} \right) \sim X^2_{n-p-1}$$

✓ Likelihood ratio chi-square test,

$$LR X^2 = -2(L_c - L_m) \sim X^2_{n-p-1},$$

where L_c is the log likelihood of model with intercept only and L_m is the log of likelihood of model with full explanatory variables.

✓ Akaike Information Criterion - the Akaike Information Criteria (AIC), can be used to compare competing models. To do this one runs the models one wishes to compare, and then compares the two models in terms of the information criterion (among other things). The higher values indicate worse fit; the preferred model is the one with the minimum value. Hence AIC not only rewards goodness of fit, but also penalizes over-fitting (increasing the number of free parameters in the model improves the goodness of the fit, regardless of the number of free parameters in the data-generating process).

$AIC = -2L_m + 2(m + 1)$, where L_m is log Likelihood of full model,

$m = \#$ of independent variables.

III. APPLICATION

A. DATA DESCRIPTION

The data used in this paper were obtained from the record of Nigeria Police force state headquarter, Ilorin, covering nine years, from 2000 to 2008. The detailed data is in the appendix1;

Y_i represents number of people killed in road accidents,

X_{1i} represents number of fatal accidents

X_{2i} represents number of serious accidents.

X_{3i} represents number of minor accidents.

a. DESCRIPTIVE STATISTICS

	N	Minimum	Maximum	Mean	Std. Deviation	Variance
Y; Persons Killed	108	0	38	12.1389	8.1510	66.4385

Table 3.1: Descriptive Statistics of counts

B. POISSON REGRESSION MODEL

POISSON REGRESSION MODEL; (R PACKAGES)

No. of obs = 144 Residual df = 107
 Deviance = 627.42 (1/df) Deviance = 5.8637
 Variance function: $V(\lambda) = \lambda$ [Poisson]

Parameter	Estimate	S.E	Z	P-value
Intercept	1.420650	0.0862	16.475	0.0000
X_1	0.1208	0.0089	13.575	0.0000
X_2	0.0422	0.0106	3.994	0.000065
X_3	0.0219	0.0095	2.311	0.0208

AIC: 834.88

Table 3.2: Poisson regression model result

C. POISSON-GAMMA REGRESSION (Using R packages)

```
glm.nb(formula, data, weights, subset, na.action, start = NULL, etastart, mustart, control = glm.control(...), method = "glm.fit", model = TRUE, x = FALSE, y = TRUE, contrasts = NULL, ... init.theta, link = log)
```

df = 107

Deviance = 189.21 (1/df) Deviance = 1.768

Generalized linear models No. of obs = 144

Parameter	Estimate	S.E	Z	P-value
Intercept	1.1825	0.1586	7.457	8.85e-14
X_1	0.1484	0.0196	7.571	3.71e-14
X_2	0.0601	0.0227	2.648	0.0081
X_3	0.0152	0.0205	0.744	0.4569

AIC: 697.8

Table 3.3: Poisson Gamma regression model result

D. COMPARISON OF THE MODELS OUTPUTS

	Poisson Estimates	Poisson Gamma Estimates
Intercept	1.42065	1.1825
S.E	0.08623	0.1586
Z-value	16.475	7.457
P-value	< 2e-16	8.85e-14
X_1 (# of fatal accidents)	0.1208	0.1484
S.E	0.0089	0.0196
Z-value	13.575	7.571
P-value	< 2e-16	3.71e-14
X_2 (# of serious accidents)	0.0422	0.0601
S.E	0.0106	0.0227
Z-value	3.994	2.648
P-value	6.5e-05	0.0081
X_3 (# of minor accidents)	0.0219	0.0152
S.E	0.0095	0.0205
Z-value	2.311	0.744
P-value	0.0208	0.4569

Table 3.4: model outputs of Poisson and Negative regression models

E. GENERAL COMPARISON OF GOODNESS OF FIT TEST FOR POISSON AND POISSON GAMMA MODELS

GOODNESS OF FIT TEST	Poisson	Poisson Gamma
Residual deviance	393.9 (df = 104)	118.2 (df = 104)
AIC	834.88	697.8

Table 3.5: goodness-of-fit tests for Poisson and Poisson Gamma regression models

Comparing the two regression models by examining the goodness-of-fit, it is clear that Poisson-Gamma regression has an edge over Poisson regression. Obviously, as good and fundamental as Poisson regression model is in GLMs, it underestimate the standard error and over-state the significance of the parameters, X_3 , due to over-dispersed, consequently giving misleading inference about the regression parameters; indicating that X_3 is significant whereas it is not (according to Poisson-Gamma Regression model, which performs better).

IV. DISCUSSION

Generalized linear models (GLMs) for count data were used because it is necessary that the count data involving number of casualty on our road cannot be used as a metric; the focus was on Poisson and Poisson-Gamma regressions. The assumption of equi-dispersion was tested, the dispersion ratio (of deviance (D)/df) is 5.8637, which is obviously greater

than 1.0, hence there is evidence of over-dispersion in the model employed; indicating that Poisson regression would not be appropriate. Instead the Poisson-Gamma regression would be more appropriate. The Deviance statistic as well as information criteria were used to test the assumptions of the underlying distributions.

V. CONCLUSION

Based on the analysis of the count data, the Poisson-Gamma model fitted the dispersed data better than the Poisson model. Though both models produced very similar coefficients; the differences are in the variances. The technique of handling dispersion parameter makes Poisson-Gamma model significantly better, which shows the strength of the model in handling over-dispersion. If over-dispersion is ignored and Poisson model used instead of Poisson-Gamma, the resulting statistical recommendation may be distorted: the Poisson model output shows that X_3 is significant while it is not (yielding type II error). Further work is on-going to examine the Conway-Maxwell Poisson regression model with the Negative Binomial regression.

However, the result of the research reflects that the number of people killed through road accident is not function of number of minor accident. Rather the number of fatal and serious cases of road accidents contributes to the number of people killed significantly.

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